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Note Title 6/5/2015

lastclass ← no caps, all one word.

Test 1 Question 1:

Base: $P(4) \cdots P(7)$

Induction Step:

For $k \geq 7$
If $P(4) \wedge P(5) \wedge \cdots P(k) \Leftrightarrow$ P(j) is true for every $j = 4, \ldots, k$.
Then $P(k+1)$

Test 1 Q4:

Base: λ, a.
Ind Step: $x \in S \rightarrow xb, xba \in S$.

length 0 λ

$\lambda b = b$
$\lambda ba = ba$

length 1 a. b.

length 2 ba. ab bb.

length 3 aba bba bab abb bbb.

Test 1 5

Prove for $n \geq 1$ 5 evenly divides $4^{2n} - 1$.

Base: $n = 1$ $4^{2 \cdot 1} - 1 = 15$

5 evenly divides 15: $5 \cdot 3 = 15$ ✓

Ind Step for $k \geq 1$ Assume 5 evenly divides $4^{2k} - 1$

Prove 5 evenly divides $4^{2(k+1)} - 1$.

$4^{2k} - 1 = 5 \cdot m$ m integer.

$4^{2k} = 5m + 1$ Int m.

$$
\begin{aligned}
4^{2(k+1)} - 1 &= 4^{2k+2} - 1 \\
&= 4^2 \, \underline{4^{2k}} - 1. \\
&= 16 \cdot (5m+1) - 1 \\
&= 5 \cdot 16 \cdot m + 16 - 1 \\
&= 5 \cdot 16m + 15 \\
&= 5(16m + 3)
\end{aligned}
$$

HW2 #12 For $n \geq 2$ $n^3 \geq 2n+4$

Base: $n=2$. $2^3 = 8 \geq 2 \cdot 2 + 4 = 8$ ✓

Assume $k \geq 2$. $k^3 \geq 2k+4$.

Prove $k \geq 1$ $k \geq 0$. $(k+1)^3 \geq 2(k+1)+4$. $= 2k+2+4 = 2k+6$

$(k+1)^3 = k^3 + 3k^2 + 3k + 1$

$\geq 2k+4 + 3k^2 + 3k+1$ (by the IH).

$\geq 2k+4 + 3k+1$ $k \geq 0$ $3k^2 \geq 0$.

$\geq 2k+4 + 3 \cdot 1 + 1$ $k \geq 1$.

$= 2k+8$

$\geq 2k+6$.

$k \geq 1$.

$3k \geq 3$

$2k+4+1+3k \geq 2k+4+1+3$

$\binom{6}{3}$

HW9 # 11. 3 - Subsets of $\{1,2,3,4,5,6\}$

$\{1,2,3\} \leq \{1,2,4\}$ $\{1,2,5\}$ $\{1,2,6\}$

$\{1,3,4\}$ $\{1,3,5\}$ $\{1,3,6\}$ $\{1,4,5\}$

$\{1,4,6\}$ $\{1,5,6\}$ $\{2,3,4\}$ - →

→ $\{4,5,6\}$

Test 1: SuperPower $(a,n) = a^{3n+1}$.

SuperPower (a,n) $\qquad a^{3\cdot 0+1} = a.$

If $(n=0)$
Return (a).

$y =$ SuperPower$(a, n-1)$

Return$(y \cdot a^3)$

$$y = a^{3(n-1)+1} = a^{3n-2}$$

$$y \cdot a^3 = a^{3n-2} \cdot a^3 = a^{3n-2+3}$$

want a^{3n+1}

Test 3 #3. Strings of length 8 over $\{a, b, c, d\}$

a) 4^8

b) a _ _ _ _ _ _ _ $\quad 4^7$

c) $\underset{3}{\bigcirc}\ \underset{3}{\bigcirc}\ \underset{4}{a}\ \underset{4}{a}\ \underset{3}{\bigcirc}\ \underset{3}{\bigcirc}\ \underset{4}{a}\ \underset{3}{\bigcirc}$ $\quad \binom{8}{3} 3^5$

d) 2 a's $\quad$ 2 c's
2 b's $\quad$ 2 d's. $\quad \dfrac{8!}{2!2!2!2!}$

$\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{2}$

e) $4^8 -$ (# of strings no c's) $= 4^8 - 3^8$.
3^8

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6. 100 Comic books (different)
4 nephews.

a). 4^{100}

b). Each nephew gets 25.

$$\binom{100}{25}\binom{75}{25}\binom{50}{25}\binom{25}{25} = \frac{100!}{25!\,25!\,25!\,25!}$$

$\underset{CB1}{4}\ \underset{CB2}{1}\ 1\ 2 \ \ldots\ \underset{CB\,100}{3}$

Test 3 #5)

a) $\binom{20}{10}$

b) Dist bins $= 20 = m$

HW passes $= 10 = n$.

$$\binom{n+m-1}{m-1} = \binom{29}{19} = \binom{29}{10}$$

c) $m = 20$

$n = 8$

$$\binom{27}{19}$$

d) ≤ 2 HW passes to Sam.

$$\binom{29}{10} - \binom{\text{\# ways Sam does not}}{\text{get } \leq 2 \text{ passes}}$$

$$\binom{\text{\# ways Sam gets}}{\geq 3 \text{ passes}}$$

$n = 7$

$m = 20$.

$$\binom{29}{19} - \binom{7+20-1}{20-1}$$

$$\binom{29}{19} - \binom{26}{19}.$$

Test 2 #9 $\qquad 7^{36} \bmod \boxed{}$

$36 = 32 + 4.$

$$7^{36} \bmod \boxed{} = 7^{32+4} \bmod \boxed{}$$

$$= 7^{32} \cdot 7^{4} \bmod \boxed{}$$

$$= (7^{32} \bmod \boxed{})(7^{4} \bmod \boxed{}) \bmod \boxed{}$$

$256 \cdot 2 \bmod \boxed{}$

$= 512 \bmod \boxed{} = 512$

Base 5 rep of 542

$(4132)_5 = 542.$

[Base 5 rep of 542 DIV 5] [542 mod 5]

$\downarrow$

[Base 5 rep of 108] 2

[21] 3

[4] 1

4