ICS 6D **Due: Monday, June 1, 2015, 2:00PM**

Homework 8

Instructor: Sandy Irani Sections 10.2, 11.1, 11.2

Leave your answer for the questions below as an arithmetic expression, including the $P(n, k)$ or $\binom{n}{k}$ notation. You do not have to compute a final numeric value.

1. For this problem you will count the number of binary strings of length 10 such that each string to be counted satisfies the following constraints:
   (a) At least one 1.
   (b) At least one 1 and at least one 0.
   (c) Exactly five 1's or begins with a 0.
   (d) The string begins or ends with a 0.
   (e) The string 01010101 appears somewhere in the string.
   (f) The number of 1's is five or it has an even number of 1's.
2. A five card hand is dealt from a standard playing deck.
   (a) How many hands have at least one 7?
   (b) How many hands have at least one Jack, Queen, or King?
   (c) How many hands have exactly two 7's or exactly two 8's?
3. A university offers 3 calculus classes: Math 2A, 2B and 2C. A set of students have each take at least one of the three classes. 51 have taken Math 2A, 80 have taken Math 2B, and 70 have taken Math 2C. 15 students have taken Math 2A and 2B, 20 have taken Math 2A and 2C, and 13 have taken Math 2B and 2C. Only 4 have taken all three classes. How many students are there in the set?
4. How many integers between 1 and 140 are integer multiples of 2, 5, or 7?
5. A family with two parents, two daughters and two sons line up for a photograph. How many ways are there for the family to line up so that the mother is next to at least one of her two daughters?
6. A coin is flipped 4 times.
   (a) What is the probability that the first flip is tails?
   (b) What is the probability that there are two consecutive heads? (You may need to do this one by just listing them).
   (c) What is the probability that the first flip is heads and there at least two consecutive heads?
7. If you have 2n socks in a drawer, n white and n black, and you reach in to choose 2 socks at random,
   (a) How many ways are there to choose?
   (b) How many of these ways result in getting a pair of the same color?
   (c) Write a simple closed form formula in terms of n for the chance choosing a matching pair of socks from a drawer with n white and n black socks.
8. 10 kids are grouped into an A team with five kids and a B team with five kids. A random assignment is chosen.

(a) What is the size of the sample space?

(b) One of the kids is named Sam and his best friend is named Mike. What is the probability that Mike and Same end up on the same team?

(c) If the group consists of five girls and five boys, what is the probability that all the girls end up on the same team?

9. A password requires that you use a sequence of upper-case characters, lower-case characters and digits. A valid password must be at least 10 characters long, and it must contain at least one character from each of the three sets of characters. If you generate 10 random characters from the union of the three sets of characters, what is the probability that you will get a valid password?

10. A 5-card poker hand from a perfectly shuffled deck is dealt to a player.

    (a) What is the probability that the hand has at least one club?

    (b) What is the probability that the hand has exactly one club or exactly one heart?

    (c) What is the probability that the hand is all from the same suit?

11. A coin is flipped ten times.

    (a) What is the probability that the first two flips are the same?

    (b) What is the probability that at least one flip comes up heads?

    (c) What is the probability that there is a run of at least 8 consecutive flips that come up the same?

    (d) What is the probability that there are two consecutive flips that are the same?

    (e) What is the probability that the number of heads is equal to the number of tails?