

1. Three propositional variables a , b , and c are defined as follows:

- a : Sam studied for his test.
- b : Sam's test was easy.
- c : Sam passed his test.

Give a logical expression that is equivalent to each sentence below using the propositional variables.

- (a) Sam passed his test, even though the test was not easy.
- (b) If Sam's test was easy or he studied for it, then he passed.
- (c) Sam passed his test only if he studied for it.

2. Define the function $f : \{0, 1\}^3 \rightarrow \{0, 1\}^3$ such that $f(x)$ is obtained from x by replacing the last bit with 1. (For example $f(110) = 111$ and $f(001) = 001$.) Is f onto? Is f one-to-one? If it is not one-to-one, give a specific example.

3. The sets A, B, C are defined as follows:

- $A = \{x \in \mathbb{Z} : x \text{ is odd}\}$
- $B = \{1, 3, 5, 7\}$.
- $C = \{2, 3, 4, 5\}$.

Circle the following expressions that are true:

- (a) $|B| = |C|$
- (b) $B \subseteq A$
- (c) $(4, 5) \in B \times C$
- (d) $A \cap C \subseteq B$.
- (e) $B \cup C = \{3, 5\}$.
- (f) $4 \in A \cup C$.

4. Define the function $g : \{0, 1\}^3 \rightarrow \{0, 1\}^3$ such that $g(x)$ is obtained from x by swapping the first bit and the last bit. (For example $g(100) = 001$.) Is g onto? Is g one-to-one? If it is not one-to-one, give a specific example.

5. Express the following using summation notation: the sum of the squares of the odd integers between 0 and 250.