1. Use strong induction to prove the following theorem by filling in the blanks for the proof.

   Theorem 1. *For any integer n such that $n \geq 18$, it is possible to make n cents worth of postage using only 4-cent or 7-cent stamps.*

   Proof:

   Bases Cases:

   Inductive Step:

   Assume that it is possible to make j cents worth of stamps for any $j =$ _____to k,

   with $k \geq$ _____.

   and prove that it is possible to make _____cents worth of stamps.

   Since $k \geq$ _____, we know by the inductive hypothesis, it is possible to make _____cents worth of stamps.

   Adding one 4-cent stamp makes _____cents worth of stamps.

   ■

2. Compute the following quantities:

   (a) $-74 \bmod 5$.

   (b) $-74 \operatorname{div} 5$

   (c) $((-59)^{27} + 87 \cdot 101) \bmod 5$.

   (d) $(26 \cdot (56 + 73)^{223}) \bmod 2$.

3. Given that $5775 = 3 \cdot 5^2 \cdot 7 \cdot 11$ and $30250 = 2 \cdot 5^3 \cdot 11^2$. Give the prime factorization for the following quantities:

   (a) $gcd(5775, 30250)$

   (b) $lcm(5775, 30250)$

   (c) $5775 \cdot 30250$