1. In class, we showed that \(NP\) can be defined to be the class of languages \(L\) such that there exists a polynomial-time decidable language \(R\) where

\[ L = \{ x \mid \exists y, |y| \leq |x|^k, (x, y) \in R \}. \]

Is it possible to define an analogous definition of the class \(NEXP\) that does not use the notion of a non-deterministic Turing Machine? Why or why not?

2. Show that if \(f(n) \geq n\) and \(g(n) \geq n\) are proper complexity functions then \(\text{TIME}(f(n)) = \text{NTIME}(f(n))\) implies that \(\text{TIME}(f(g(n))) = \text{NTIME}(f(g(n)))\).

3. Define a \textit{coding} \(\kappa\) to be a mapping from \(\Sigma\) to \(\Sigma\). Note that \(\kappa\) need not be one-to-one. If \(x = \sigma_1 \ldots \sigma_n\), where each \(\sigma_i \in \Sigma\), then we define \(\kappa(x) = \kappa(\sigma_1) \ldots \kappa(\sigma_n)\). If \(L\) is a language, then \(\kappa(L)\) is defined to be \(\{ \kappa(x) \mid x \in L \}\).

(a) Prove that \(NP\) is closed under codings. That is, show that if \(L \in NP\) and \(\kappa\) is a coding defined on the alphabet of \(L\), then \(\kappa(L) \in NP\).

(b) We expect that \(P\) is not closed under codings, but we can not prove this without establishing that \(P \neq NP\). Instead, show that \(P\) is closed under codings if and only if \(P = NP\).