Do two of the three problems below.

1. An input to the problem ST-NON-CONN is a graph $G$ along with vertices $s$ and $t$ in $G$. The language ST-NON-CONN consists of all triplets $(G, s, t)$ such that there is not a path from $s$ to $t$ in $G$. Prove that ST-NON-CONN is complete for the class co-NL.

2. In the Arora-Boak text gives an alternative definition of the class NL which makes use of a Turing Machine with a special read-once tape. The head on a read-once tape starts at the left-most end of the non-blank symbols written on the tape and can only move to the right or stay in the same place (i.e. it can never move left). The alternative definition says that a language $L$ is in NL if there is a deterministic Turing Machine $M$ (called a verifier) with a special read-once tape and a polynomial $p$ such that for every $x \in \Sigma^*$,

   \[
   x \in L \iff \exists u \in \Sigma^{p(|x|)} \text{ such that } M(x, u) = 1,
   \]

   where $M(x, u)$ is the output of $M$ when $x$ is placed on the input tape and $u$ is placed on its special read-once tape and $M$ uses $O(\log n)$ space on its work tape for every input $x$.

   Prove that this definition is equivalent to the definition using non-deterministic Turing Machines discussed in class.

3. Prove that if in the above definition, the read-once tape is replaced with a read-only tape (which could be read many times), then the resulting class is NP.