Do three of the following five problems:

1. Show that the class $\text{ZPP} = \text{RP} \cap \text{co-RP}$.

2. Describe a decidable language that is in $\text{P/poly}$ but not in $\text{P}$.

3. We have been careful to have gates with bounded fan-in (that is, they each take at most two inputs). However, we have allowed unbounded fan-out in that each gate can feed into an arbitrary number of gates. Show that any circuit can be transformed into another with fan-out two with only a linear increase in the size.

4. The language $\text{USAT}$ is the set of boolean formulae that have a unique satisfying assignment. In class we proved the Valiant-Vazirani theorem which says that that there exists a polynomial-time algorithm $f$ such that for every $n$-variable boolean formula, $\phi$

$$
\begin{align*}
\phi \in \text{SAT} & \Rightarrow Pr[f(\phi) \in \text{USAT}] \geq \frac{1}{8n} \\
\phi \notin \text{SAT} & \Rightarrow Pr[f(\phi) \in \text{USAT}] = 0.
\end{align*}
$$

Now suppose we have a polynomial time algorithm that given a boolean formula $\phi$, will answer ”yes” if $\phi \in \text{USAT}$ , will answer ”no” if $\phi \notin \text{SAT}$ and will answer arbitrarily otherwise. Prove that this would imply that $\text{RP} = \text{NP}$.

5. A language $L \subseteq \{0, 1\}^*$ is sparse if there is a polynomial $p$ such that $|L \cap \{0, 1\}^n| \leq p(n)$ for all $n$. Show that every sparse language is in $\text{P/poly}$.