Section 1.1 piazza.com/uci/winter2015/ics6b

Note Title 1/4/2015

A proposition is a statement that is either true or false.

Examples: $2+3=7$ F

$5-2=3$ T

7 is a prime number T

Today is Friday F

It will rain tomorrow ?

Vanilla is the best flavor of ice cream.

Not propositions:

How are you?

Eat your vegetables.

Logical variables: $(p, q, r, \ldots)$

denote an arbitrary proposition
truth value can be true or false

true: $p = T$
false: $p = F$.

like variables from algebra except value is T or F, instead of a number.

Logical operations can be used to combine propositions to get compound propositions.

Conjunction "AND" Symbol $\wedge$

If p & q are propositional variables,

$p \wedge q$ is a proposition.
truth value of $p \wedge q$ depends on truth values of p & q.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

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truth table shows truth value of $p \wedge q$ for every possible truth value for p and q.

p : Sam is poor
q : Sam is happy

Different ways to express $p \wedge q$ in English!

Sam is poor, but he is happy.
Sam is poor and happy
Although Sam is poor, he is happy.

Disjunction: $p \lor t$ p or t

p	t	$p \lor t$
T	T	T
T	F	T
F	T	T
F	F	F

"inclusive" or.

Ambiguity in English:

Tonight I will go to the party or I will go to a movie.

The patient has high blood pressure or has a history of migraines.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

exclusive or.

Negation: $\neg p$.

p: it is raining today

$\neg p$: it is not raining today

it is not true that it is raining today

p	$\neg p$
T	F
F	T

Review:

$p = T$

$q = F$

$r = T$.

$p \wedge q$ F

$r \wedge p$ T

$q \vee r$ T

$p \oplus r = F$.

$p \vee r$ T

$\neg p$ F

$\neg q$ T

Section 1.2

Note Title 1/6/2015

Compound propositions can be built using one or more logical operation:

$p \vee \neg r$ $\quad$ $p = F$, $r = T$

F F → F

Need to specify the order in which operations are performed:

$p \wedge q \vee r$ $\quad$ $p = F$, $q = T$, $r = T$

$(p \wedge q) \vee r = T$

$p \wedge (q \vee r) = F$

Order in which logical operations are applied:

1. $\neg$
2. $\wedge$
3. $\vee$

$\neg q \vee r$ $\quad$ $\neg(q \vee r)$ $\quad$ $(\neg q) \vee r$

$(p \wedge q) \vee \neg t$

Can override the default order with parens:

$$\neg(q \vee r)$$

$r = T$
$q = F$.

$$\neg(\underbrace{F \quad T}) $$

$$\neg T \Rightarrow F$$

Good to include parens as a reminder:

$$(p \wedge q) \vee r$$

$$\neg p \vee (t \wedge r)$$

$$\neg T \quad \underbrace{F \quad T}$$

$p = T$
$t = F$
$r = T$.

$$(F \vee F) \Rightarrow F.$$

$$\neg(p \wedge t \wedge r)$$

$$T \quad T \quad F$$

$p = T$
$t = T$
$r = F$

$$\neg F \Rightarrow T.$$

A truth table for a compound proposition shows the truth value for every possible combination of truth values for the propositional variables:

$\neg p$	p	q	$\neg p \vee q$
F	T	T	T
F	T	F	F
T	F	T	T
T	F	F	T

p	q	r	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \wedge r$
T	T	T	F	T	T
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	T	T	T
F	F	F	T	T	F

If a compound proposition has n variables the truth table has 2^n rows.

2 variables $\Rightarrow$ $2^2 = 4$
3 variables $\Rightarrow$ $2^3 = 8$

p: $\pi > 3$

q: 3 is a root of the equation $x^2 - 2 = 0$.

r: The integer 5 is even.

$\neg r$ The integer 5 is odd.

$\neg p \vee q$

$\neg p$: $\pi \leq 3$

$\pi \leq 3$ or 3 is a root of the eqn. $x^2 - 2 = 0$.

$q \wedge r$

$\neg (q \wedge r)$ It's not the case that $\pi > 3$ and the integer 5 is even

Section 1.3

Conditional operation

p, q propositions $\qquad p \to q$ false only when p is true and q is false.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p is the hypothesis
q is the conclusion

$\underline{p} \to q \qquad\qquad p \to q$

p : you study hard
q : you will get an A

If you study hard, then you will get an A.

Ways to express in English: $p \to q$.

- if p then q.
- if p, q
- p implies q.
- q, if p.
- p only if q
- p is sufficient for q.
- q is necessary for p.

You have your D.L. only if you are at least 16 years old.

p : You have a driver's license.
q : You are at least 16 years old.