Section 1.3 Conditional operation

Note Title 1/7/2015

p, q propositions $\quad$ $p \to q$ false only when p is true and q is false.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p is the hypothesis
q is the conclusion

$p \to q$ $\qquad$ $p \to q$

p : you study hard
q : you will get an A

If you study hard, then you will get an A.

Ways to express in English: $p \to q$.

- if p then q.
- if p, q
- p implies q.
- q, if p.
- p only if q
- p is sufficient for q.
- q is necessary for p.

You have your D.L. only if you are at least 16 years old

p : You have a driver's license.
q : You are at least 16 years old.

Statement: $\underline{p \to q}$.

Converse: $q \to p$

⇒ Contrapositive: $\neg q \to \neg p$

Inverse: $\neg p \to \neg q$.

p	q	$p \to q$	$\neg q \to \neg p$ (contrapositive)	$q \to p$	$\neg p \to \neg q$
T	T	T	T	T	T
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	T	T	T

Statement: $s \longrightarrow m$

if Sally will join the club th Mildred joins the club. *

Identify the converse, contrapositive inverse.

$\neg s \longrightarrow \neg m$ Inverse

⇒ If Sally does not join the club then Mildred won't either.

$m \longrightarrow s$ Converse

If Mildred joins the club, then so will Sally.

$\neg m \longrightarrow \neg s$ Contrapos. *

If Mildred doesn't join the club then Sally won't join the club.

Biconditional :

$\underline{p \leftrightarrow q}$ — p if and only if q.

p iff q.

p is necessary and sufficient for q.

$p \leftrightarrow q$ means $\underline{(p \to q)} \wedge \underline{(q \to p)}$

p and q have the same truth value

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

p if & only if q

p only if q — $p \longrightarrow q$

p if q — $p \longleftarrow q$

$q \to p$

p if q

p only if q

Truth Values for:

T — If $\overset{T}{3 \text{ is prime}}$ then $\overset{T}{4 \text{ is even}}$.

F — If $\overset{T}{3 \text{ is prime}}$ then $\overset{F}{5 \text{ is even}}$

T — If $\overset{F}{4 \text{ is prime}}$ then $\overset{F}{5 \text{ is even}}$.

T — If $\overset{F}{4 \text{ is prime}}$ then $\overset{T}{4 \text{ is even}}$.

p if q

$q \to p$.

T — $\overset{T}{\pi > 3}$ if and only if $\overset{T}{\sqrt{5} > 2}$.

Conditional operations in compound propositions:

$(p \to q) \vee r$?

$p \to (q \vee r)$

$(p \to q) \to r$

$p \to (q \to r)$

Order: $\neg$, $\wedge$, $\vee$, $\{\to, \leftrightarrow\}$ → use parens anyway!

Truth table for $\neg(p \vee q) \to r$

p	q	r	$p \vee q$	$\neg(p \vee q)$	
T	T	T	T	F	T
T	T	F	T	F	T
T	F	T	T	F	T
T	F	F	T	F	T
F	T	T	T	F	T
F	T	F	T	F	T
F	F	T	F	T	T
F	F	F	F	T	F

From English to logic:

c : I will return to college
j : I will get a job.

$\neg j$ c

Not getting a job is sufficient for me to return to college.

$$\neg j \to c.$$

If I return to college, then I won't get a job.

$$c \to \neg j$$

I am not getting a job but I am still not returning to college.

$$\neg j \wedge \neg c$$

I will return to college only if I won't get a job.

$$c \to \neg j$$

There is no way I am returning to college.
$\neg c$

I will get a job and return to college.

$$j \wedge c$$

s: a student is a senior
v: a student is at least 17 years old.
p: a student is allowed to park in the parking lot.

A student is allowed to park in the parking lot only if they are a senior and at least 17 years old.

$$p \rightarrow (s \wedge v)$$

A student can park in the parking lot if they are a senior or at least 17 years old.

$$(s \vee v) \rightarrow p$$

(Section 1.4) Logical Equivalence

Note Title 1/8/2015

Two compound propositions are logically equivalent if they have the same truth value for every combination of truth values for the propositional variables in the compound expressions.

Example: $p \to q$ $\quad \neg q \to \neg p$

p	q	$p \to q$	$\neg q \to \neg p$	$q \vee \neg p$
T	T	T	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Notation $\quad p \to q \equiv \neg q \to \neg p$

if $p = T$

then $\neg p = F$.

p	q	$p \wedge (p \vee q)$
T	T	T
T	F	T
F	T	F
F	F	F

Two logically equivalent compound propositions don't necessarily have the same set of variables.

$$p \equiv p \wedge (p \vee q).$$

de Morgan's $\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$.

Another example.

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	T	F	F
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

A proposition is a tautology if it is always true, regardless of the truth value of its variables. $(\equiv T)$

A proposition is a contradiction if it is always false, regardless of the truth value of its variables. $(\equiv F)$.

p	q	$p \to q$	$\neg p \to (p \to q)$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

tautology

contradiction

p	q	$p \wedge q$	$\neg p \vee \neg q$	$(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$
T	T	T	F	F
T	F	F	T	F
F	T	F	T	F
F	F	F	T	F

Can use simple logical equivalence facts to establish more complex ones.

de Morgan's law: $\boxed{\neg(p \wedge q) \equiv \neg p \vee \neg q}$

$$(p \vee r) \rightarrow \neg(p \wedge q) \equiv (p \vee r) \rightarrow (\neg p \vee \neg q)$$

$$\neg(\underbrace{(s \rightarrow t)}_{p} \wedge \underbrace{(t \vee r)}_{q}) \equiv \neg(s \rightarrow t) \vee \neg(t \vee r)$$

$$\neg p \vee \neg q$$