

Section 1.3

Conditional operation

Note Title

1/7/2015

P, q propositions

$P \rightarrow q$

false only when P is true and q is false.

	P	q	$P \rightarrow q$
\Rightarrow	T	T	T
	T	F	F
	F	T	T
	F	F	T

P is the hypothesis
 q is the conclusion

$P \rightarrow q$

$P \rightarrow q$

P : you study hard
 q : you will get an A

If you study hard, then you will get an A.

Ways to express in English: $P \rightarrow q$.

if P then q .

if P, q

P implies q .

q , if P .

\Rightarrow P only if q \leftarrow

\Rightarrow P is sufficient for q .

\Rightarrow q is necessary for P .

You have your D.L. only if you are at least 16 years old

P : You have a driver's license. \leftarrow

q : You are at least 16 years old. \leftarrow

Statement: $p \rightarrow q$.

Converse: $q \rightarrow p$

\rightarrow Contrapositive: $\neg q \rightarrow \neg p$

Inverse: $\neg p \rightarrow \neg q$.

Contrapositive

P	q	$P \rightarrow q$	$\neg q \rightarrow \neg p$	$q \rightarrow p$	$\neg p \rightarrow \neg q$
T	T	T	T	T	T
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	T	T	T

Red arrows indicate that the columns for $P \rightarrow q$ and $\neg q \rightarrow \neg p$ are identical, as are the columns for $q \rightarrow p$ and $\neg p \rightarrow \neg q$.

Statement: $S \rightarrow M$
if Sally will join the club then Mildred joins the club. *

Identify the converse, contrapositive inverse.

$\neg S \rightarrow \neg M$ Inverse

\Rightarrow If Sally does not join the club then Mildred won't either.

$M \rightarrow S$ Converse

If Mildred joins the club, then so will Sally.

$\neg M \rightarrow \neg S$ Contrapos. *

If Mildred doesn't join the club then Sally won't join the club.

Biconditional : $p \leftrightarrow q$ p if and only if q .
 p iff q .
 p is necessary and sufficient for q .

$p \leftrightarrow q$ means $(p \rightarrow q) \wedge (q \rightarrow p)$

p and q have the same truth value

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

p if and only if q

p only if q $p \rightarrow q$
 p if q $p \leftarrow q$
 $q \rightarrow p$
 p if q
 p only if q

Truth values for:

T If 3 is prime then 4 is even.

F If 3 is prime then 5 is even.

T If 4 is prime then 5 is even.

T If 4 is prime then 4 is even.

T $\pi > 3$ if and only if $\sqrt{5} > 2$.

p if q
 $q \rightarrow p$

Conditional operations in compound propositions:

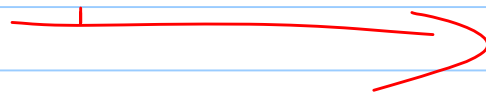
$$(p \rightarrow q) \vee r \quad ?$$

$$p \rightarrow (q \vee r)$$

$$(p \rightarrow q) \rightarrow r$$

$$p \rightarrow (q \rightarrow r)$$

Order: $\neg, \wedge, \vee, \{ \rightarrow, \leftrightarrow \}$ → use parens anyway!



Truth table for $\neg(p \vee q) \rightarrow r$

p	q	r	$p \vee q$	$\neg(p \vee q)$	$\neg(p \vee q) \rightarrow r$
T	T	T	T	F	T
T	T	F	T	F	F
T	F	T	T	F	T
T	F	F	T	F	F
F	T	T	T	F	T
F	T	F	T	F	F
F	F	T	F	T	T
F	F	F	F	T	F

From English to logic:

c : I will return to college

j : I will get a job.

Not getting a job is sufficient for me to return to college.

$$\neg j \rightarrow c$$

If I return to college, then I won't get a job.

$$c \rightarrow \neg j$$

I am not getting a job but I am still not returning to college.

$$\neg j \wedge \neg c$$

I will return to college only if I won't get a job.

$$c \rightarrow \neg j$$

There is no way I am returning to college.

$$\neg c$$

I will get a job and return to college.

$$j \wedge c$$

S: a student is a senior

V: a student is at least 17 years old.

P: a student is allowed to park in the parking lot.

A student is allowed to park in the parking lot only if they are a senior and at least 17 years old.

$$P \rightarrow (S \wedge V)$$

A student can park in the parking lot if they are a senior or at least 17 years old.

$$(S \vee V) \rightarrow P$$

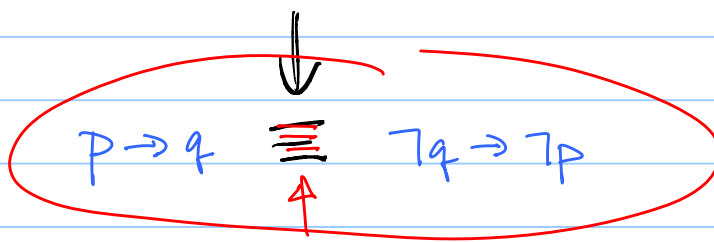
(Section 1.4) Logical Equivalence

Two compound propositions are logically equivalent if they have the same truth value for every combination of truth values for the propositional variables in the compound expressions.

Example:

P	q	$P \rightarrow q$	$\neg q \rightarrow \neg P$	$q \vee \neg P$
T	T	T	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Notation



P	q	$P \wedge (P \vee q)$
T	T	T
T	F	T
F	T	F
F	F	F

if $P = T$
then $\neg P = F$.

Two logically equivalent compound propositions don't necessarily have the same set of variables.

$$P \equiv P \wedge (P \vee q).$$

de Morgan's

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q.$$

Another example.

P	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	T	F	F
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

↓

↑

A proposition is a tautology if it is always true, regardless of the truth value of its variables.
($\equiv T$)

A proposition is a contradiction if it is always false, regardless of the truth value of its variables.
($\equiv F$).

P	q	$p \rightarrow q$	$\neg p \rightarrow (p \rightarrow q)$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

} → tautology

} → contradiction

P	q	$p \wedge q$	$\neg p \vee \neg q$	$(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$
T	T	T	F	F
T	F	F	T	F
F	T	F	T	F
F	F	F	T	F

Can use simple logical equivalence facts to establish more complex ones.

de Morgan's law:

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$(p \vee r) \rightarrow \neg(p \wedge q) \equiv (p \vee r) \rightarrow (\neg p \vee \neg q)$$

$$\neg \left(\underbrace{(s \rightarrow t)}_p \wedge \underbrace{(t \vee r)}_q \right) \equiv \neg(s \rightarrow t) \vee \neg(t \vee r)$$
$$\qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \neg p \vee \neg q$$