Note Title 1/12/2015

Show: $\neg(p \vee q \vee r) \equiv \neg p \wedge \neg q \wedge \neg r$.

$\neg(p \vee q \vee r)$

$\neg(\underbrace{(p \vee q)} \vee \underbrace{r})$ Assoc Law.

$\neg(p \vee q) \wedge \neg r$ De Morgan's Law.

$(\neg p \wedge \neg q) \wedge \neg r$ "

$\neg p \wedge \neg q \wedge \neg r$ Assoc.

Show: $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$.

$\neg(p \vee (\neg p \wedge q))$

$\neg p \wedge \neg(\neg p \wedge q)$ DM

$\neg p \wedge (\neg\neg p \vee \neg q)$ DM

$\neg p \wedge (p \vee \neg q)$ Involution.

$(\neg p \wedge p) \vee (\neg p \wedge \neg q)$ Distr.

$\underline{F} \vee \underline{(\neg p \wedge \neg q)}$ Complement

$(\neg p \wedge \neg q) \vee F$ Comm.

$(\neg p \wedge \neg q)$ Id.

$p \wedge (q \vee r) \not\equiv (p \wedge q) \vee r$.

Show: $(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r)$
$\equiv (p \wedge \neg r)$.

$$(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r)$$

$$((p \wedge \neg r) \wedge q) \vee ((p \wedge \neg r) \wedge \neg q) \quad \text{Comm.}$$

$$((p \wedge \neg r) \wedge q) \vee ((p \wedge \neg r) \wedge \neg q) \quad \text{Assoc.}$$

$$(p \wedge \neg r) \wedge (q \vee \neg q) \quad \text{Dist.}$$

$$(p \wedge \neg r) \wedge T \quad \text{Compl.}$$

$$(p \wedge \neg r) \quad \text{Id.}$$

Show: $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.
$\equiv T$.

$$(p \wedge q) \rightarrow (p \vee q)$$

$$\neg (p \wedge q) \vee (p \vee q) \quad \text{Cond Id.}$$

$$(\neg p \vee \neg q) \vee (p \vee q) \quad \text{DM.}$$

$$(\neg p \vee \neg q \vee p \vee q) \quad \text{Assoc.}$$

$$(\neg p \vee p \vee \neg q \vee q) \quad \text{Comm}$$

$$((\neg p \vee p) \vee \neg q \vee q) \quad \text{Assoc.}$$

$$(T \vee (\neg q \vee q)) \quad \text{Compl.}$$

$$T \quad \text{id.}$$

Section 1.5 Predicates.

Statement: x is prime.

Not a proposition.
Truth value depends on value of x.

If $x = 5$: "5 is prime" true.

If $x = 4$: "4 is prime" false

A Predicate is a Statement whose truth value depends on the values of one or more variables.

$P(x)$: x is prime.

$P(5)$: 5 is prime
$P(4)$ 4 is prime.
] propositions.

In defining $P(x)$ need to specify domain of variable x. The set of all possible values for x.

Domain for x in $P(x)$ positive integers.

Predicate $P(x)$ not a proposition.
if n is a specific value in the domain of x
then $P(n)$ is a proposition.

Predicates can have more than one variable:

$Q(x,y)$: $x^2 = y$.
$R(x,y,z)$: $x^2 + y^2 = z^2$.

(Domain all integers) for x, y, z.

$\Rightarrow Q(-5, 25)$ $(-5)^2 = 25$ ✓ TRUE.
$\Rightarrow Q(5, -25)$. False.

$\cancel{Q(5, y)}$ not a prop.

$R(3, 4, 7)$: $3^2 + 4^2 = 7^2$ False
$R(3, 4, 5)$. $3^2 + 4^2 = 5^2$ True.

Predicates aren't always about numbers:

Domain: Student ID's for all students enrolled in Winter 2015 ICS 6B.

$T(x)$: x is a transfer student.
$C(x)$: x is a computer science major.

$T(30410221)$
$C(52107916)$
↳ propositions!

One way to make a predicate into a proposition is to plug in specific values.

Another way is to use quantifiers:

Universal Quantifier : $\forall x\, P(x)$ For all x, $P(x)$

Existential Quantifier : $\exists x\, P(x)$ There exist an x such that $P(x)$.

If the domain for x is finite: $\{a_1, a_2, \ldots, a_n\}$

$\Rightarrow$ $\forall x\, P(x) \equiv P(a_1) \land P(a_2) \land \ldots \land P(a_n)$

$\exists x\, P(x) \equiv P(a_1) \lor P(a_2) \lor \ldots \lor P(a_n)$.

$\forall x\, T(x)$ is only true if everyone in ICS 6B is a transfer student.

If one student is not a transfer student it is false.

$\exists x\, T(x)$ is true if at least one student is a transfer student.

False only if all students are not transfer students.

Domain positive integers.

$\exists x \; x^2 = x$ ($x=1$, $1^2=1$) $\Rightarrow$ True.

$\forall x \; x^2 = x$. Counter example $x=2$ $2^2 \neq 2$.
$\Rightarrow$ False.

$\forall x \; |x| = x$ True.

$\exists x \; x^2 > x$ $x=2$ $2^2 > 2$ $\Rightarrow$ True.

$\exists x \; x^2 < x$ False. To prove false
for every pos int, $x^2 \not< x$.

Can use quantifiers & logical operations together

$\forall x \, (P(x) \wedge Q(x))$ $\forall x \, (T(x) \wedge C(x))$

parentheses important! (free & bound variables).

"bound"

$\forall x \; P(x) \wedge Q(x) \equiv (\forall x \; P(x)) \wedge Q(x)$

$\forall x \, T(x) \wedge C(x)$ "free".

$\exists x \, T(x) \wedge \exists x \, C(x)$

$\exists x \, (T(x) \wedge C(x))$