

Show that $\exists x (P(x))$ is false

$\exists x \neg P(x)$

Domain: Set of kids in a class.

$C(x)$: x has a cat

$D(x)$: x has a dog.

$\exists x (C(x) \wedge D(x))$

$C(\text{Sam}) \wedge D(\text{Sam})$

	$C(x)$	$D(x)$
Sam	T	T
Suzy	F	F
Ben	F	F

$\exists x C(x) \wedge \exists x D(x)$

$= \exists x C(x) \wedge \exists y D(y)$

$T \wedge T$

$= T$

$\Rightarrow C(\text{Ben}) \rightarrow D(\text{Ben})$ T

$C(\text{Suzy}) \vee D(\text{Sam})$ F

$C(x)$ $D(x)$

Bill	T	F
Frank	F	T
Margie	T	F

$\exists x C(x) \wedge \exists x D(x)$

$= \exists x C(x) \wedge \exists y D(y)$

$C(\text{Bill}) \wedge D(\text{Frank})$
 $T \wedge T = T$

Someone has a dog and someone has a cat.

$\Rightarrow \exists x (C(x) \wedge D(x))$

$= (C(\text{Bill}) \wedge D(\text{Bill})) \vee (C(\text{Frank}) \wedge D(\text{Frank})) \vee (C(\text{Margie}) \wedge D(\text{Margie}))$
 $F \vee F \vee F = F$

Someone has a dog and a cat.

$\hookrightarrow F$

Domain: employees at a company

$T(x)$: x is an executive.

$B(x)$: x received a large bonus.

Someone did not get a large bonus.

$$\exists x \neg B(x)$$

Everyone got a large bonus.

$$\forall x B(x)$$

Sam did not receive a large bonus even though he is an executive.

$$\neg B(\text{Sam}) \wedge T(\text{Sam})$$

* There is an executive who received a large bonus.

$$\exists x (T(x) \wedge B(x))$$

$$\forall x P(x)$$

$$\rightarrow \exists x P(x).$$

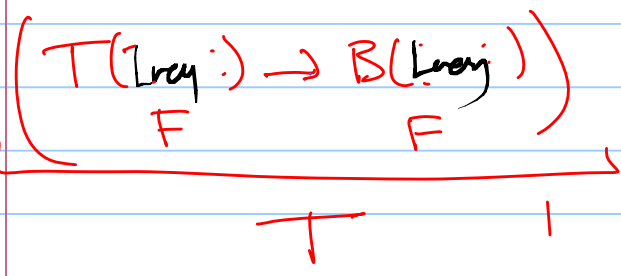
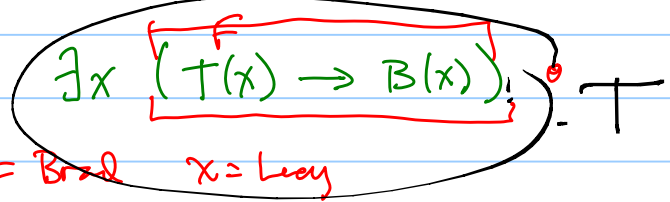
~~X~~

* Every executive got a large bonus.

$$\forall x (T(x) \rightarrow B(x))$$

	T(x)	B(x)
Joe	T	F
Belinda	T	F
Lucy	F	F
Brad	F	T

"There is an executive who got a large bonus."



$\exists x (T(x) \wedge B(x)) \cdot F$

	T(x)	B(x)
Joe	T	T
Belinda	T	T
Lucy X	F	F
Brad X	F	T

"Every executive got a large bonus."

$\forall x (T(x) \wedge B(x))$

counterexamples: Lucy & Brad

	$T(x) \rightarrow B(x)$
Joe	T
Belinda	T
Lucy	T
Brad	T

$\forall x (T(x) \rightarrow B(x))$

T

De Morgan's Law for Quantified Statements.

$$\neg \exists x T(x) \equiv \forall x \neg T(x).$$

There does not exist a student in the class who is a transfer student.

Every student in the class is not a transfer student.

Consistent w/ DM's law on a finite domain:

$$\neg (T(a_1) \vee T(a_2) \dots \vee T(a_n)) \equiv \neg T(a_1) \wedge \neg T(a_2) \wedge \dots \wedge \neg T(a_n)$$

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$$\neg \exists x T(x) \qquad \qquad \qquad \forall x \neg T(x).$$

$$\neg \forall T(x) \equiv \exists x \neg T(x)$$

It's not true that all the students are transfer students.

There is at least one student who is not a transfer student.

What is $\neg \forall x (x^2 > x) \equiv \exists x \neg (x^2 > x)$

$$\equiv \exists x (x^2 \leq x)$$

$$\neg \exists x (x^2 = 2)$$

$$\forall x \neg (x^2 = 2)$$

$$\forall x (x^2 \neq 2)$$

Show: $\neg \forall x (P(x) \rightarrow Q(x)) \equiv \exists x (P(x) \wedge \neg Q(x))$

True for any domain & truth values.

$$\neg \forall x (P(x) \rightarrow Q(x))$$

Can't do this w/ truth tables.

$$\exists x \neg (P(x) \rightarrow Q(x)) \quad \text{DM.}$$

$$\exists x \neg (\neg P(x) \vee Q(x)) \quad \text{Cond Id.}$$

$$\exists x \neg \neg P(x) \wedge \neg Q(x) \quad \text{DM.}$$

$$\exists x P(x) \wedge \neg Q(x) \quad \text{Inv. (?)}$$

Predicates w/ more than one variable:

$$\begin{array}{ll} \exists y \forall x S(x,y) & \text{prop.} \\ \exists y \forall z \forall x Q(x,y,z) & \text{prop.} \end{array}$$

To make into propositions, need to have a quantifier for each variable.

Nested Quantifiers of the Same type

Domain: set of people at a meeting.

$H(x,y)$: x shook y 's hand.

$$\forall x \forall y H(x,y)$$

"Everyone shook everyone's hand."

Includes the case $x=y$!

$$\forall x \forall y (x \neq y \rightarrow H(x,y))$$

"Everyone shook everyone else's hand."

$$x=y=Sam.$$

$$(Sam \neq Sam) \rightarrow H(Sam, Sam).$$

T.

$P(x,y)$: x knows y 's phone number.

$\exists x \exists y P(x,y)$

"Someone knows someone's phone number."

could be satisfied by the case $x=y$.

$\exists x \exists y (x \neq y \wedge P(x,y))$

"Someone knows someone else's phone number."

Nested Quantifiers

$\exists x \forall y P(x,y)$

Someone knows everyone's phone number.

$\forall x \exists y P(x,y)$

Everyone knows someone's phone number.

Game between two players:

Existential Player

trying to make the predicate true

Selects existentially quantified variables

Predicate true?

Quant Statement is true

Universal Player

trying to make the predicate false.

Selects universally quantified variables.

Predicate false?

Quant statement false.