Show that $\exists x\,(P(x))$ is false

$\exists x\, \neg P(x)$

Domain: Set of kids in a class.

$C(x)$: x has a cat

$D(x)$: x has a dog.

	$C(x)$	$D(x)$
Sam	T	T
Suzy	F	F
Ben	F	F

$\exists x\,(C(x) \land D(x))$

$C(\text{Sam}) \land D(\text{Sam})$

$\underbrace{\exists x\, C(x) \land \exists x\, D(x)}$

$= \underbrace{\exists x\, C(x)}_{T} \land \underbrace{\exists y\, D(y)}_{T}$

$T \land T$

$= T$

$\Rightarrow \underbrace{\overset{F}{C(\text{Ben})} \to \overset{F}{D(\text{Ben})}}$ T

$\underbrace{\overset{F}{C(\text{Suzy})} \lor \overset{T}{D(\text{Sam})}}$ T

	$C(x)$	$D(x)$
Bill	T	F
Frank	F	T
Margie	T	F

$\exists x\, C(x) \land \exists x\, D(x)$

$\equiv \underbrace{\exists x\, C(x)}_{T} \land \underbrace{\exists y\, D(y)}_{T}$

$C(\text{Bill})$ $D(\text{Frank})$

$T \land T \to T$

Someone has a dog and someone has a cat.

$\Rightarrow \exists x\,(C(x) \land D(x))$

$\equiv \underbrace{(\overset{T}{C(\text{Bill})} \land \overset{F}{D(\text{Bill})})}_{F} \lor \underbrace{(\overset{F}{C(\text{Frank})} \land \overset{T}{D(\text{Frank})})}_{F} \lor \underbrace{(\overset{T}{C(\text{Margie})} \land \overset{F}{D(\text{Margie})})}_{F}$

$\hookrightarrow$ F.

Someone has a dog and a cat.

Domain: employees at a company

$T(x)$: x is an executive.
$B(x)$: x received a large bonus.

Someone did not get a large bonus.

$$\exists x \neg B(x)$$

Everyone got a large bonus.

$$\forall x \, B(x)$$

Sam did not receive a large bonus even though he is an executive.

$$\neg B(Sam) \wedge T(Sam)$$

* There is an executive who received a large bonus.

$$\exists x \, (T(x) \wedge B(x))$$

$\forall x \, P(x) \rightarrow \exists x \, P(x)$

$\cancel{\leftarrow}$

* Every executive got a large bonus.

$$\forall x \, (T(x) \rightarrow B(x))$$

	T(x)	B(x)
Joe	T	F
Belinda	T	F
Lucy	F	F
Brad	F	T

"There is an executive who got a large bonus." F

$$\exists x \underbrace{(T(x) \to B(x))}_{F} \quad T$$

x = Brad   x = Lucy

equiv.

$$\exists x\,(T(x) \wedge B(x)) \quad F$$

$$\big(\underset{F}{T(\text{Lucy})} \to \underset{F}{B(\text{Lucy})}\big)$$

T

	T(x)	B(x)
Joe	T	T
Belinda	T	T
Lucy ✗	F	F
Brad ✗	F	T

[Every executive got a large bonus.] T

$$\forall x\,(T(x) \wedge B(x)) \quad F$$

Counterexamples: Lucy & Brad

$$\forall x\,(T(x) \to B(x))$$

T

	$T(x) \to B(x)$
Joe	T
Belinda	T
Lucy	T
Brad	T

Note Title 1/12/2015

De Morgan's Law for quantified statements.

$$\neg \exists x\, T(x) \equiv \forall x\, \neg T(x).$$

There does not exist a student in the class who is a transfer student.

Every student in the class is not a transfer student.

Consistent w/ DM's law on a finite domain:

$$\neg\,(T(a_1) \vee T(a_2) \cdots \vee T(a_n)) \equiv \neg T(a_1) \wedge \neg T(a_2) \wedge \cdots \wedge \neg T(a_n)$$

$$\neg \exists x\, T(x) \qquad\qquad \forall x\, \neg T(x).$$

$$\neg \forall\, T(x) \equiv \exists x\, \neg T(x)$$

It's not true that all the students are transfer students.

There is at least one student who is not a transfer student.

What is $\neg \forall x\, (x^2 > x) \equiv \exists x\, \neg(x^2 > x)$

$\equiv \exists x\, (\underbrace{x^2 \leq x})$

$\neg \exists x\, (x^2 = 2)$ $\quad \forall x\, \neg(x^2 = 2)$

$\forall x\, (x^2 \neq 2)$

Show: $\neg \forall x\, (P(x) \to Q(x)) \equiv \exists x\, (P(x) \wedge \neg Q(x))$

$\boxed{\text{True for any domain \& truth values.}}$

$\neg \forall x\, (P(x) \to Q(x))$

$\boxed{\text{Can't do this w/ truth tables.}}$

$\exists x\, \neg(P(x) \to Q(x))$ DM.

$\exists x\, \neg(\neg P(x) \vee Q(x))$ Cond Id.

$\exists x\, \neg\neg P(x) \wedge \neg Q(x)$ DM.

$\exists x\, P(x) \wedge \neg Q(x)$ Inv. (?).

Predicates w/ more than one variable:

$\forall y \exists x\, S(x,y)$ prop.

$\exists y\ \forall z\ \forall x\, Q(x,y,z)$ prop.

To make into propositions, need to have a quantifier for each variable.

Nested Quantifiers of the Same type

Domain: set of people at a meeting.

$H(x,y)$: x shook y's hand.

$\forall x \forall y\, H(x,y)$ "Everyone shook everyone's hand".

Includes the case $x=y$!

$\forall x \forall y (x \neq y \rightarrow H(x,y))$ "Everyone shook everyone else's hand."

$x = y = \text{Sam}$.

$$\underbrace{(\overset{F}{\text{Sam} \neq \text{Sam}}) \rightarrow H(\text{Sam}, \text{Sam})}_{T}.$$

$P(x,y)$: x knows y's phone number.

$\exists x \exists y \, P(x,y)$ "Someone knows someone's phone number."

could be satisfied by the case $x=y$.

$\exists x \exists y \, (x \neq y \wedge P(x,y))$ ← "Someone knows someone else's phone number."

Nested Quantifiers

$\exists x \forall y \, P(x,y)$ Someone knows everyone's phone number.

$\forall x \exists y \, P(x,y)$ Everyone knows someone's phone number.

Game between two players:

Existential Player	Universal Player
trying to make the predicate true	trying to make the predical false.
Selects existentially quantified variables	Selects universally quantified variables.
Predicate true?	Predicate false?
Quant Statement is true	Quant statement false.