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Note Title 1/14/2015

$P(x,y)$: x knows y's phone number.

$\exists x \, \exists y \, P(x,y)$ "Someone knows some one's phone number."

could be satisfied by the case $x=y$.

"Someone knows someone else's phone number."

Nested Quantifiers

$\exists x \, \forall y \, P(x,y)$ Someone knows everyone's phone number.

$\forall x \, \exists y \, P(x,y)$ Everyone knows someone's phone number.

Game between two players:

Existential Player	Universal Player
trying to make the predicate true	trying to make the predical false.
Selects existentially quantified variables	Selects universally quantified variables.
Predicate true?	Predicate false?
Quant Statement is true	Quant statement false.

Domain = {1, 2, 3, 4, 5}

$\exists x\, \forall y\, \underbrace{P(x,y)}_{T.}$ — Someone knows everyone's phone number.

True

	y = 1	2	3	4	5
x = 1	F	F	F	F	F
2	F	T	F	F	T
3	T	T	T	T	T
4	F	T	T	T	T
5	T	T	T	T	F

$x=3$

False

	y = 1	2	3	4	5
x = 1	T	F	T	T	T
2	F	T	F	F	T
3	T	F	T	F	F
4	F	T	T	T	T
5	T	T	T	T	F

F

$x=1$

$\forall x\, \exists y\, P(x,y)$ — True

Everyone knows someone's phone number.

Domain: all real numbers.

$\forall x\, \exists y \quad x + y = 0.$ $\quad y = -x \quad$ True.

$\exists x\, \forall y. \quad x + y = 0.$ $\quad y = -x+1$ (for example)

$x+y = x + (-x+1) = 1 \neq 0$

$\forall x\, \exists y \quad x \cdot y = 1.$ $\quad x = 0.$ False

$\exists x\, \forall y \quad x \cdot y = 0.$ $\quad x = 0$ True.

$\forall x\, \exists y\, \left((x = 0) \vee x \cdot y = 1\right)$ ⇐ Domain = real #'s ~~integers.~~

If $x = 0$ T

$x \neq 0$ $\quad y = 1/x$

Domain: Set of students in a class

$P(x,y)$: x knows y's phone number.

$K(x)$: x has the answer key.

Someone who has the answer key knows everyone's phone number.

There is a person who has the answer key and knows everyone's phone number.

$$\exists x\, (\underbrace{K(x) \wedge}_{\text{no}\ -\ y} \forall y\, P(x,y))$$

$$\exists x\, \forall y\, (K(x) \wedge P(x,y))$$

Everyone who has the answer key knows someone's phone number.

$$\forall x\, (K(x) \rightarrow \exists y\, P(x,y) \quad)$$

$$\forall x\, \exists y\, (K(x) \rightarrow P(x,y)).$$

Everyone who has the answer key knows someone else's phone number.

$$\forall x\, (K(x) \rightarrow (\exists y\, ((x \neq y) \wedge P(x,y))))$$

Domain: runners in a race.

$A(x)$: x is an adult.
$C(x)$: x is a child.
$B(x,y)$: x beat y.

An adult won the race.

There is an adult who beat everyone else.

There is someone who is an adult who beat everyone else.

$$\exists x \left(A(x) \wedge \forall y \left((x \neq y) \rightarrow B(x,y) \right) \right)$$

There is a child who beat every adult in the race.

$$\exists x \left(C(x) \wedge \forall y \left(A(y) \rightarrow B(x,y) \right) \right)$$

Every adult beat every child in the race.

$$\forall x \forall y \left((A(x) \wedge C(y)) \rightarrow B(x,y) \right)$$

There is a child who beat an adult in the race.

$\exists x \left(B(Sam, x) \wedge \forall y \left((x \neq y) \to \neg B(Sam, y) \right) \right)$

There is someone that Sam beat, and for everyone else Sam did not beat them.

Sam beat exactly one person in the race.

Sam beat at least one person in the race.

$\exists x \; B(Sam, x)$.

Sam beat exactly two people in the race:

Cannot do: $\forall A(x) \exists y \; B(x,y)$

$\forall x \exists y \; B(C(x), A(x))$.

De Morgan's Law with nested quantifiers:

When the negation moves from the left to the right of a quantifier, the quantifier changes: $\forall \to \exists$
$\exists \to \forall$.

Find a logically equivalent quantified statement in which negation signs immediately precede a predicate:

$$\neg \forall x \exists y \forall z \, P(x,y,z).$$

$$\equiv \exists x \neg \exists y \forall z \, P(x,y,z)$$

$$\exists x \forall y \neg \forall z \, P(x,y,z)$$

$$\exists x \forall y \exists z \neg P(x,y,z).$$

$$\neg \forall x \exists y \, (D(x) \to P(x,y)).$$

$$\exists x \forall y \neg (D(x) \to P(x,y))$$

$$\exists x \forall y \neg (\neg D(x) \vee P(x,y))$$

$$\exists x \forall y (\neg\neg D(x) \wedge \neg P(x,y))$$

$$\exists x \forall y (D(x) \wedge \neg P(x,y))$$

$$\neg \exists x \forall y \, (T(x) \wedge S(x,y))$$