Note Title 1/20/2015

Argument:

$$\left.\begin{array}{c} p_1 \\ p_2 \\ \vdots \\ p_n \end{array}\right\} \text{hypotheses}$$

$$\underline{\qquad\qquad}$$

$\therefore\ c$ conclusion.

$\therefore$ means "therefore".

An argument is valid if the conclusion is true whenever all the hypotheses are true.

Argument is valid if

* $(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \to c$ is a tautology

For example:

p 1

$p \to q$ 2

$\therefore q.$ c

p	q	$p \to q$	$(p \wedge (p \to q))$	$(p \wedge (p \to q)) \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Here's an example of an argument that is not valid

$p \to q$ T

$\neg p$ T

$\therefore \neg q$ F

$((p \to q) \wedge \neg p) \to \neg q$ is not a tautology.

$p = F$

$q = T$

Arguments are often expressed in English as in:

(1) If $\overbrace{\text{I will study for my exam}}^{p}$, then $\overbrace{\text{I will pass my exam}}^{q}$.
(2) ~~I will study for my exam~~. p.
$\therefore$ I will pass my exam.

The **form** of an argument expressed in English is obtained by replacing each distinct proposition with a propositional variable:

- p : I will study for my exam.
  q : I will pass my exam.

$$\begin{array}{l} p \to q \\ \underline{p \qquad\qquad} \\ \therefore \; q. \end{array}$$

An argument can be invalid even if the hypotheses & the conclusion are true:

If $\overbrace{\text{5 is even}}^{p}$, then $\overbrace{\text{7 is even}}^{q}$. T
5 is not even $\neg p$ T
$\therefore$ 7 is not even. $\neg q$ T

p : 5 is even
q : 7 is even

$$\begin{array}{l} p \to q \\ \underline{\neg p \qquad\qquad} \\ \therefore \neg q. \end{array}$$

In order for an argument to be valid, it must be true regardless of the actual truth values of the propositional variables.

This same argument could be used to conclude something that is not true:

If 4 is a prime number then 5 is a prime number.
4 is not a prime number

∴ 5 is not a prime number.

p: 4 is prime
q: 5 is prime.

$$\begin{array}{l} p \to q \\ \neg p \\ \hline \therefore \neg q. \end{array}$$

Another example:

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q. \end{array}$$

Valid argument.

p	q	$p \vee q$	$\neg p$	$(\neg p \wedge (p \vee q)) \to q$
T	T	T	F	T
T	F	T	F	T
F	T ?	T	T	T
F	F	F	T	T

An alternative way to show an argument is valid: apply arguments that have already been shown to be valid.

(Much like showing logical equivalences using laws of propositional logic which are themselves logical equivalences already established via truth tables.)

There is a set of rules of inference that are useful for putting together valid arguments. (Go through them).

$$\begin{array}{ll} * & p \to q \\ * & q \to r \\ & \neg r \\ \hline & \therefore \neg p. \end{array}$$

1.	$q \to r$	Hyp.
2.	$\neg r$	Hyp.
3.	$\neg q$	Mod Toll on 1+2.
4.	$p \to q$	Hyp.
5.	$\neg p$	Mod Toll. on 3+4

Sometimes in reasoning it is clearer to translate English sentences into the language of logic and use rules of inference to reason.

If I have a hard workout, I am be sore. If I am sore I take asperin. I am not taking asperin.