Note Title 1/21/2015

There is a set of rules of inference that are useful for putting together valid arguments. (Go through them).

$p \to q$
$q \to r$
$\neg r$
$\therefore \neg p$.

1) $p \to q$ hyp
2) $q \to r$ hyp
3) $p \to r$ Hyp syll. 1,2
4) $\neg r$ hyp
5) $\neg p$ Modus Tollens. 3,4

Sometimes in reasoning it is clearer to translate English sentences into the language of logic and use rules of inference to reason.

If I have a hard workout, I am be sore. If I am sore I take asperin. I am not taking asperin.

w: work out
s: sore.
a: asprin

$w \to s$
$s \to a$
$\neg a$
$\therefore \neg w$.

If its not foggy (f) or it doesn't rain (r) then the race (c) will be held and we will go see the race (g).
If the race is held, there will be a trophy ceremony.
The trophy ceremony was not held.
∴ It rained

• also using identities from before.

$$(\neg f \vee \neg r) \rightarrow (c \wedge g)$$
$$c \rightarrow t$$
$$\neg t$$
$$\therefore r$$

1) $c \rightarrow t$ hyp
2) $\neg t$ hyp
3) $\neg c$ Mod Tol. 1,2
4) $\neg c \vee \neg g$ addition 3
5) $\neg (c \wedge g)$ De Morgan. 4
6) $(\neg f \vee \neg r) \rightarrow (c \wedge g)$ hyp
7) $\neg(\neg f \vee \neg r)$ Mod Toll. 5,6
8) $\neg\neg f \wedge \neg\neg r$ De.M. 7
9) $f \wedge r$ Double Negation. 8
10) r Simplification 9

This kind of argument is particularly useful in using quantifiers.

Universal Instantiation

$$\frac{\forall x\, P(x)}{\therefore P(c)}$$

c is any element in the domain.

Every student in the class got an A

∴ Sam (a student in the class) got an A.

Universal Generalization

"arbitrary" means nothing is assumed about c other than the fact that it is in the domain.

$$\frac{P(c) \text{ for an arbitrary } c \text{ in the domain}}{\therefore \forall x\, P(x)}$$

Let x be an arbitrary real number:

$1 > 0$

$x+1 > x+0 = x$

$\therefore \forall x \; x+1 > x$

Domain: real numbers.

Existential Intantiation

Caution! Use a "fresh" variable every time this rule is used.

$$\frac{\exists x\, P(x)}{\therefore P(c) \text{ for some } c \text{ in the domain.}}$$

Some student in the class got an A.

∴ "c" got an A where c is in the class.

Existential Generalization

$$\frac{P(c) \text{ for some } c \text{ in the domain}}{\therefore \exists x\, P(x)}$$

Sam a student in the class got an A

Some student in the class got an A.

Linda, an employee of the company, owns a Ferrari. $F(x)$ x owns a Ferrari.
Everyone who owns a Ferrari has gotten a speeding ticket. $S(x)$ gets a speeding ticket
∴ A student in the class has gotten a speeding ticket.
$\exists x\ (C(x) \wedge S(x))$

Univ students in class.

$F(\text{Linda}) \wedge C(\text{Linda})$
$\forall x\ (F(x) \to S(x))$
$F(\text{Linda}) \to S(\text{Linda})$
$S(\text{Linda})$

$\therefore\ \exists x\ (S(x) \wedge C(x))$

Univ: all people

$C(x)$ x is class.

1) $F(\text{Linda}) \wedge C(\text{Linda})$. hyp.
2) $\forall x\ (F(x) \to S(x))$. hyp.
3) $F(\text{Linda})$ Simp 1.
4) $F(\text{Linda}) \to S(\text{Linda})$ Univ Inst.
5) $S(\text{Linda})$. Mod Pon 3,4
6) $C(\text{Linda})$ Simp 1.
7) $S(\text{Linda}) \wedge C(\text{Linda})$. Addition 5,6
8) $\exists x\ (S(x) \wedge C(x))$ Exist gen 7

Univ: all paintings.

$M(x)$ Matisse painted x.
$H(x)$ x is in my house.
$B(x)$ x is beautiful.

$\forall x\,(M(x) \to B(x))$
All of Matisse's paintings are beautiful.
I have a Matisse painting in my house. $\exists x\,(M(x) \wedge H(x))$
$\therefore$ There is a beautiful painting in my house. $\exists x\,(H(x) \wedge B(x))$

1)	$\exists x\,(M(x) \wedge H(x))$	hyp
2)	$M(p) \wedge H(p)$.	Exist Inst 1.
3)	$M(p)$	Simpl 2
4)	$\forall x\,(M(x) \to B(x))$	hyp.
5)	$M(p) \to B(p)$.	Univ. Inst. 4.
6)	$B(p)$	Mod Pon 3,5
7)	$H(p)$.	Simpl 2.
8)	$B(p) \wedge H(p)$.	Addition 7,8.
9)	$\exists x\,(B(x) \wedge H(x))$	Exist Gen 8.

Common Pitfalls: $\exists x\, P(x) \not\to P(\text{Sam})$.

Argument

$\exists x\, P(x)$
$\exists x\, Q(x)$
$\therefore \exists x\,(P(x) \wedge Q(x))$

Invalid?

Proof:

$\exists x\, P(x)$
$P(c)$
$\exists x\, Q(x)$
$Q(c)$
$P(c) \wedge Q(c)$.
$\exists x\,(P(x) \wedge Q(x))$

not necessarily the same c.

To show an argument w/ quantifiers is not valid
give an example of a domain + values
for the predicates that make all the
hypotheses true and the conclusion false.

Domain = $\{c, d\}$.

	$P(x)$	$Q(x)$
$x = c$	T	F
$x = d$	F	T

$\exists x\, P(x)$
$\exists x\, Q(x)$ are both true
but $\exists x\, (P(x) \wedge Q(x))$
is not true.