Note Title 2/8/2015

Constructing a Boolean expression that computes the same function as defined in an input/output table.

x	y	z	f(x,y,z)	
0	0	0	1	$\bar{x}\bar{y}\bar{z}$
0	0	1	0	
0	1	0	0	
0	1	1	1	$\bar{x}yz$
1	0	0	0	
1	0	1	0	
1	1	0	1	$xy\bar{z}$
1	1	1	1	xyz

$$f(x,y,z) = \bar{x}\bar{y}\bar{z} + \bar{x}yz + xy\bar{z} + xyz$$

Boolean expressions can be put in standardized forms. (Useful for reasoning about them or manipulating them systematically).

Disjunctive Normal Form (DNF)

Sum of product of literals.

Conjunctive Normal Form (CNF)

Product of sums of literals.

Our method for converting I/O tables to Boolean expressions produces expressions in DNF form!

$$\underbrace{x\bar{y}z} + \underbrace{\bar{x}\bar{y}z} + \underbrace{\bar{x}\bar{y}\bar{z}}$$

The product terms need not be minterms. The following Boolean expression are all in DNF form:

$$\underbrace{x\bar{y}z} + \underbrace{\bar{x}} + \underbrace{y\bar{z}} \qquad \underbrace{x} + \underbrace{y} + \underbrace{z}$$

$$\underbrace{\bar{u}xz} + \underbrace{\bar{z}\bar{x}} + \underbrace{y} \qquad \underbrace{xyz}$$

A Boolean Expression is in DNF form iff multiplication only applied to literals.

Not in DNF form:

$(x+y)z + \bar{x}\bar{y}\bar{z}$

$z(x+y)u$

CNF Boolean expressions look like:

$(\bar{x}+\bar{y}+\bar{z})(x+y)(\bar{u}+x)$ $\quad$ $(u+x+z)(y+\bar{x})$

$x(\bar{y}+\bar{z})u$

xyz $\quad$ $x+y+z$

A Boolean Expression is in CNF form iff the addition operator is only applied to literals:

Not in CNF Form:

$(x+y+uz)(\bar{x}+\bar{y})\bar{z}$ $\quad$ $xy+z$

Does every Boolean function have a representation as a CNF Boolean expression?

Yes! Here is a systematic way for finding it:

Will use DeMorgan's law!

$$\overline{(u+v+w+x+y+z)} \equiv \bar{u}\,\bar{v}\,\bar{w}\,\bar{x}\,\bar{y}\,\bar{z}$$

$$\overline{uvwxyz} = \bar{u}+\bar{v}+\bar{w}+\bar{x}+\bar{y}+\bar{z}$$

x	y	z	f(x,y,z)	$\overline{f(x,y,z)}$
0	0	0	1	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	0	1
1	1	0	1	0
1	1	1	1	0

$$\overline{f(x,y,z)} = \underbrace{\bar{x}\bar{y}z}_{} + \underbrace{\bar{x}y\bar{z}}_{} + \underbrace{x\bar{y}\bar{z}}_{} + \underbrace{x\bar{y}z}_{}$$

$$f(x,y,z) = \overline{\overline{f(x,y,z)}} = \overline{\overline{f(x,y,z)}}$$

$$f(x,y,z) = \overline{\underbrace{\bar{x}\bar{y}z}_{a} + \underbrace{\bar{x}y\bar{z}}_{b} + \underbrace{x\bar{y}\bar{z}}_{c} + \underbrace{x\bar{y}z}_{d}}$$

$$(\overline{\bar{x}\bar{y}z})(\overline{\bar{x}y\bar{z}})(\overline{x\bar{y}\bar{z}})(\overline{x\bar{y}z})$$

$$(x+y+\bar{z})(x+\bar{y}+z)(\bar{x}+y+z)(\bar{x}+y+\bar{z})$$