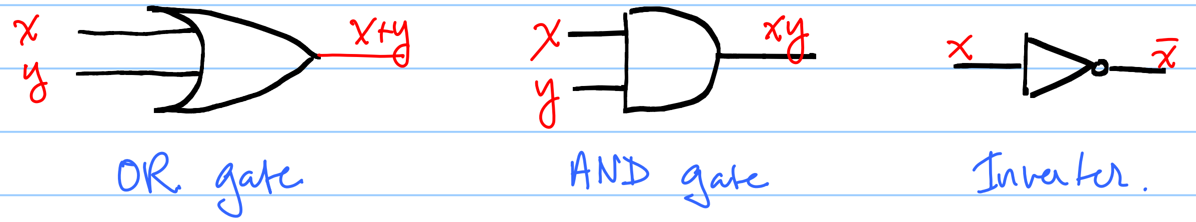
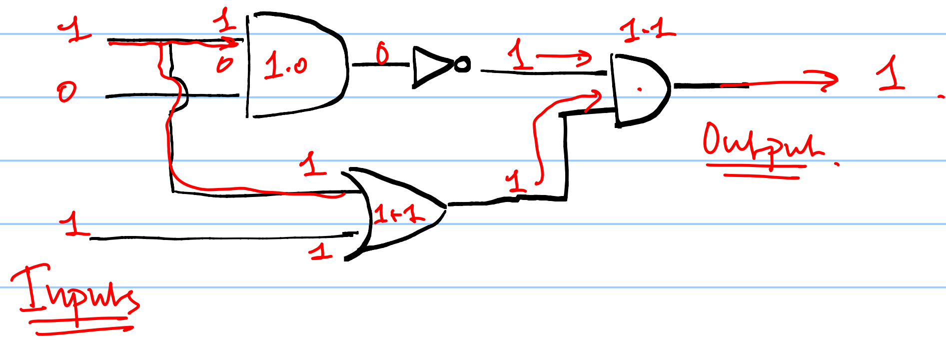
Gates & Circuits.

Note Title 2/13/2015

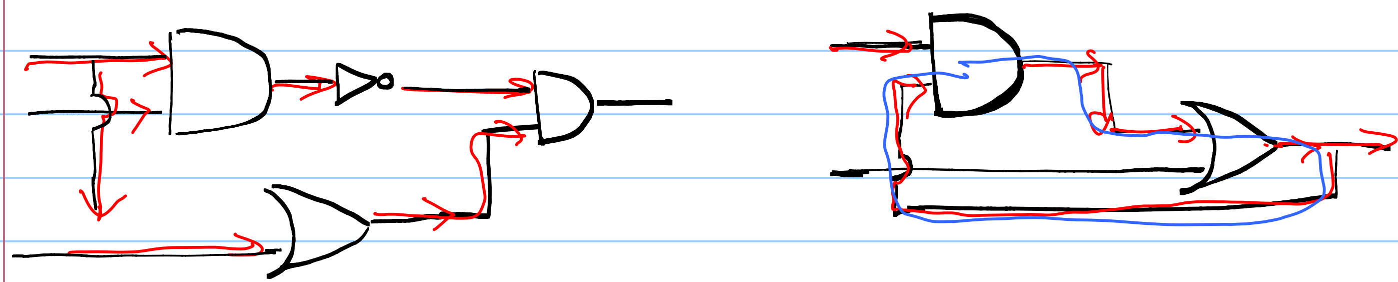
Circuits to compute Boolean functions are built from gates.

3 different types of gates to compute the three Boolean operations: addition, multiplication, complement.



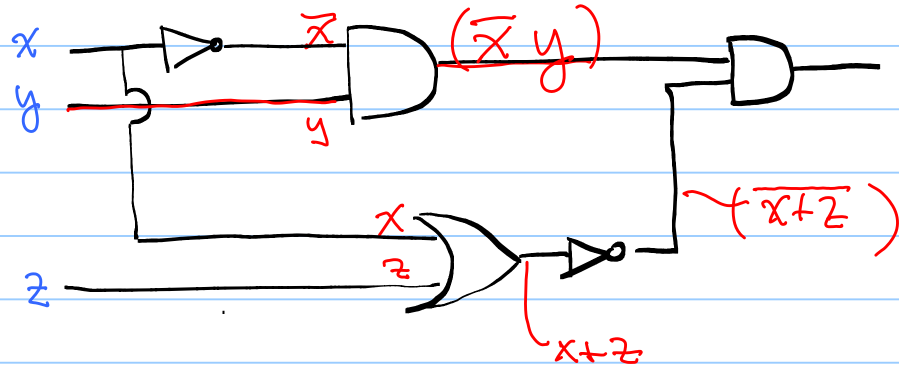


We will only look at combinatorial circuits:
Output depends only on the input variables not on the state of the circuit.
(No cycles)



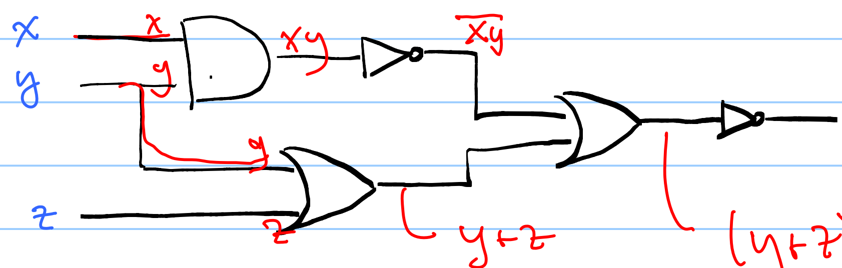
Boolean Circuit $\Rightarrow$ Boolean Expression

Boolean Circuit $\Leftarrow$ Boolean Expression



$(\bar{x}y)(\overline{x+z})$

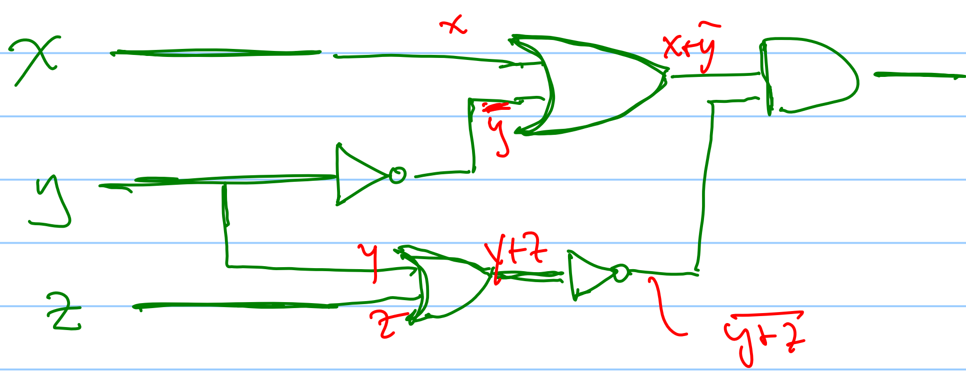
$= \bar{x}y(\overline{x+z})$



$\overline{y+z+\overline{xy}}$

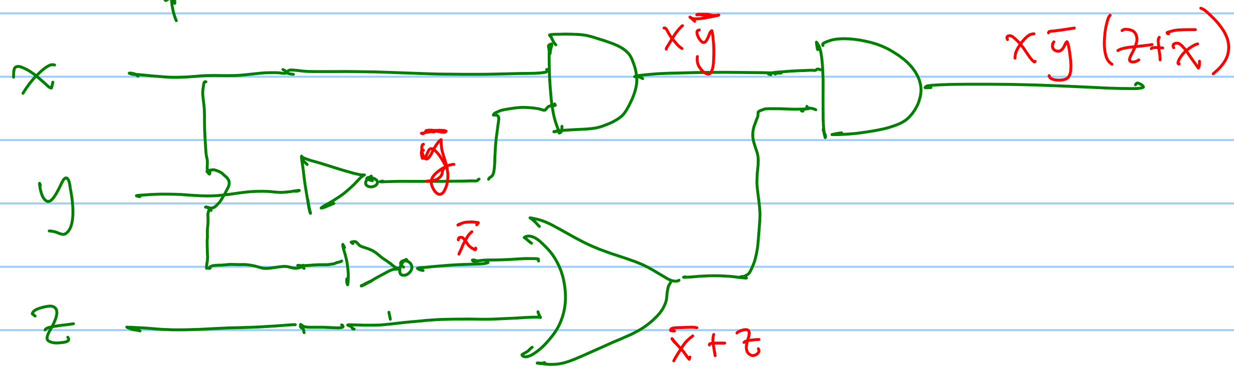
$(y+z)+(\overline{xy})$

$(x+\bar{y})(\overline{\bar{y}+z})$



$\overline{y+z}$

$x\bar{y}(z+\bar{x})$



Circuit Design:

English Description

$\Downarrow$

Input/Output Table

$\Downarrow$

Boolean Expression

$\Downarrow$ → Simplification.

Boolean Circuit

Binary Relations.

Note Title 2/13/2015

Set A & B   two sets.

Binary Relation on/between A & B is a Subset of $\underline{A \times B}$.

$\Rightarrow R \subseteq A \times B$ $(a,b) \notin R$

$a \in A$
$b \in B$.

$\underline{(a,b) \in R} \iff \underline{aRb}$. $a \not R b$

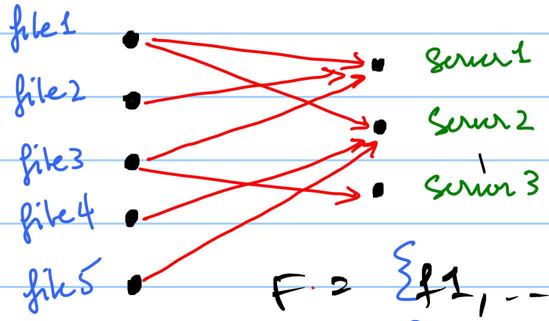
Example: Files in a large database stored in a distributed network.

F = set of all files.
S = set of all servers in the network.

Relation R to express whether a particular file f stored on a particular server s.

fRs f stored on s.

There can be more than one copy of a particular file stored on different servers (for fault tolerance).



$F = \{f1, \ldots, f5\}$
$S = \{s1, s2, s3\}$

$R = \{(f1, s1), (f1, s2), (f2, s1), (f3, s1), (f3, s3), (f4, s2), (f5, s2)\}$
$\subseteq F \times S$.

$R(x, y)$

A relation R between sets F + S is mathematically the same as a predicate w/ two input variables:

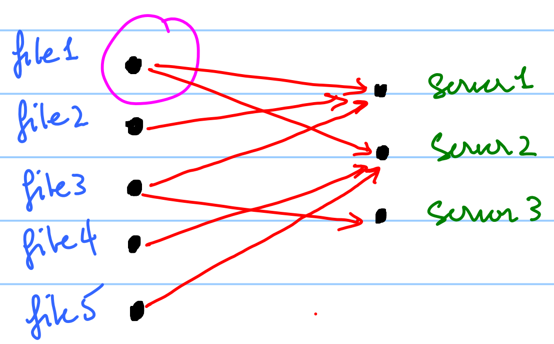
F domain of x + S domain of y:

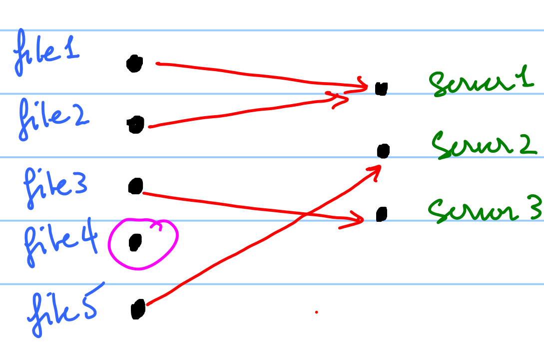
$$R(f,s) = \text{true} \quad \text{iff} \quad (f,s) \in R.$$

A Relation is a generalization of a function.

Every function $f: F \to S$ is a relation between F + S

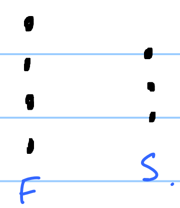
Some relations between F + S are not well defined functions.





These are both relations but not functions.

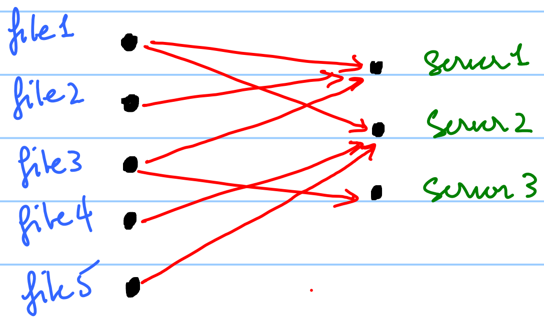
A Relation can be empty:



$$R \subseteq F \times S$$

$$R = \emptyset$$

A relation on finite sets can be represented by a matrix:



$$\rightarrow \begin{array}{c} \\ \text{file 1} \\ \text{file 2} \\ \text{file 3} \\ \text{file 4} \\ \text{file 5} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Serv 1} & \text{Serv 2} & \text{Serv 3} \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

$R \subseteq F \times S$:

rows $\longrightarrow$ elements in F

columns $\rightarrow$ elements in S.

Relations can be between infinite sets:

Relation C between $\mathbb{Z}$ and $\mathbb{R}$. $\quad C \subseteq \mathbb{Z} \times \mathbb{R}$.

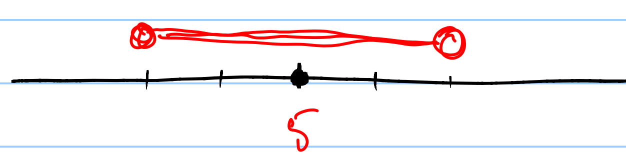
$x \, C \, y$ iff $|x-y| < 2$.

$3 \not{C} \, 5.2 \qquad |3-5.2| \overset{?}{\not<} 2 \quad$ (2.2)

$3 \, C \, 1.1$ yes

$-4 \, C \, -5.5$ yes $\qquad |-4 - -5.5| \overset{?}{<} 2$

$5 \, C \, y$?



Relation R between a set A & itself is a subset of $A \times A$.

"R is a binary relation on the set A".
A is the domain of R.

Examples: Relation P on $\mathbb{N}$.

aPb iff there is an $n \in \mathbb{N}$ such that $b = a^n$
"b is a power of a".

$3P9$ yes $\quad 9 = 3^2$

$9 \not P 3$ $\quad 9^{\square} = 3$

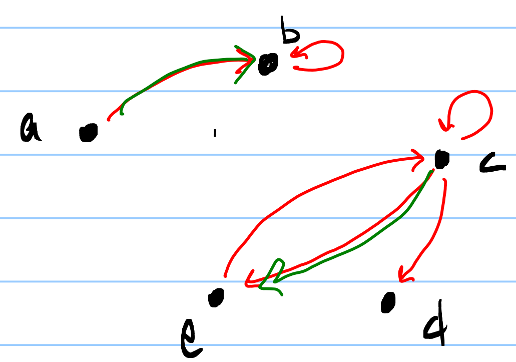
$2P16$ yes $\quad 2^4 = 16$

$2 \not P 20$.

$4 = 4^1$

$4P4$ yes.

$A = \{a, b, c, d, e\}$



R.

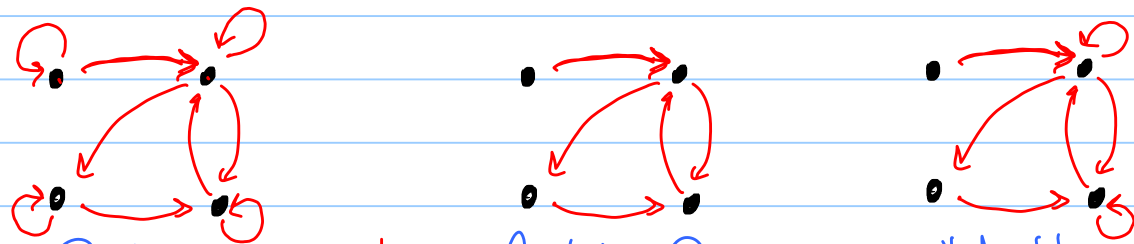
$R = \{(a,b), (b,b), (c,c), (e,c), (c,e), (c,d)\}$

Properties of Relations:

Relation R on A

Reflexive: $\forall a \in A, \quad (a,a) \in R$.

Anti-Reflexive: $\forall a \in A, \quad (a,a) \notin R$.



Reflexive   Anti-R   Neither

S = a set of people.

Relation: M sMt if s & t have the same birthday.

Reflexive

E sEt if s earns more money than t.

Anti-reflexive

M sMt if s sent an email to t yesterday.

Relation R on $\mathbb{R}$. xRy if $|x| = y$.

$3R3 \quad |3| = 3$

$-4 \not R -4 \quad |-4| \neq -4$.

Neither.