Note Title

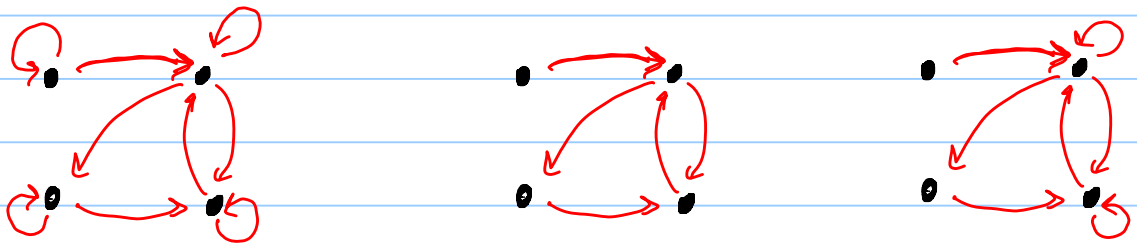
Properties of Relations:

Relation R on A

$R \subseteq A \times A$.

Reflexive: $\forall\, a \in A, \quad (a,a) \in R$.

Anti-Reflexive: $\forall\, a \notin A, \quad (a,a) \notin R$.



S = a set of people.

Relation: M $\quad sMt$ if $s \neq t$ have the same birthday.

E $\quad sEt$ if s earns more money than t.

M $\quad sMt$ if s sent an email to t yesterday.

Relation R on $\mathbb{R}$. $\quad xRy$ if $|x| = y$.

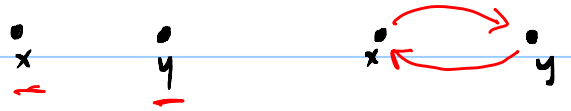
2/13/2015

Symmetric / Anti-Symmetric. Relation R on A.

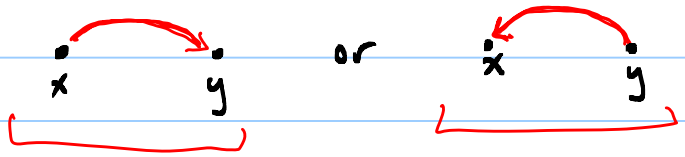
Symmetric: for every $x, y \in A$ $\quad xRy \rightarrow yRx$

(trivially true if $x=y$.)

For $x \neq y$:



Never have:

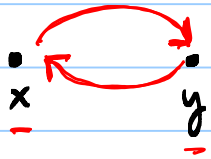


Anti-Symmetric: For every $x, y \in A$

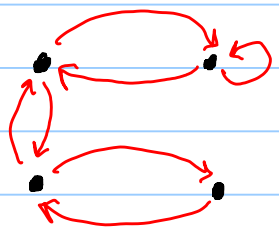
xRy and $yRx \rightarrow x=y$.

equivalent to xRy and $x \neq y \rightarrow y \not R x$

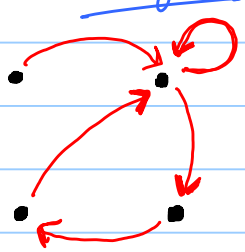
Can never have:



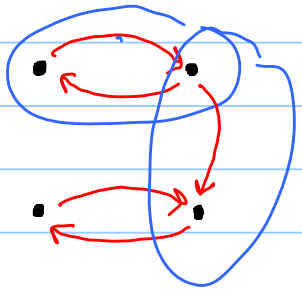
symmetric



anti-sym



neither



P on $\mathbb{Z}^+$ $\quad xPy$ iff $\exists\ n \in \mathbb{N}$ $\quad x^n = y$. $\quad \rightarrow$ Anti-Symmetric

$3P9$ $\quad 3^2 = 9$.

$9 \not P 3$ $\quad$ not symmetric

$x^n = y$ $\quad n \geq 1$.

$y^m = x$?

$y^m = x$.

$xPy \wedge yPx \Rightarrow x=y$.

anti-Symmetric

L on $\mathbb{R}$ $\quad x L y$ iff $x < y$.

if $x < y \wedge y < x \Rightarrow x = y$.

E on $\mathbb{R}$ $\quad x E y$ if $x \leq y$. anti-symmetric

$\exists E \subseteq \mathbb{R}^3$.

if $x \leq y \wedge y \leq x \Rightarrow x = y$.

S on $\mathbb{R}$ $\quad x S y$ iff $x^2 < y$.

?

$1/3 \; S \; 1/4$ $\quad (1/3)^2 < 1/4$

$1/4 \; S \; 1/3$ $\quad (1/4)^2 < 1/3$

neither

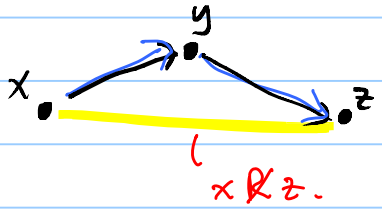
$2 \; S \; 5$ $\quad 2^2 < 5$

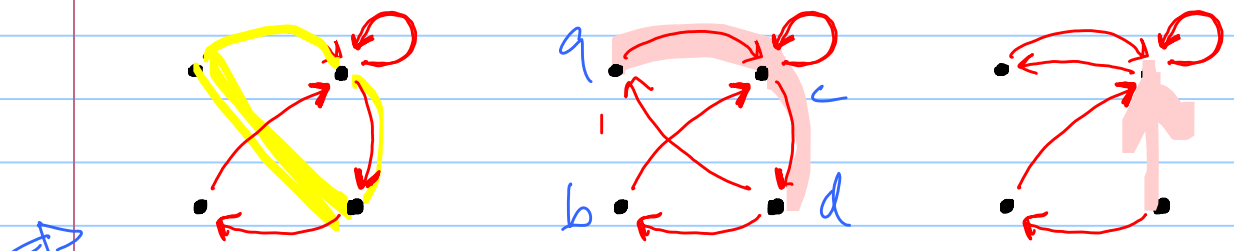
$5 \not S \; 2$. $\quad 5^2 < 2$

A relation R on set S is transitive if

xRy and $yRz \Rightarrow xRz$.

Never have:





not transitive.

$aRc \wedge cRd$.

but $a \not R d$. → not transitive.

P on $\mathbb{Z}$ $\quad xPy$ iff $\exists\, n \in \mathbb{N}$ $\quad x^n = y$.

Yes

$xPy \wedge yPz$

$y = x^n$

$z = y^m$

$x^{\square} = z$ (exponent: nm)

?

$z = y^m = (x^n)^m = x^{nm}$

M sMt if s & t have the same birthday.

Trans!

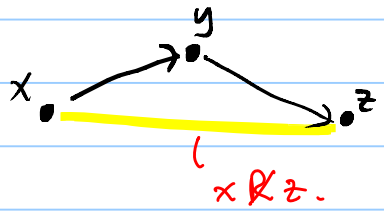
E sEt if s earns more money than t.

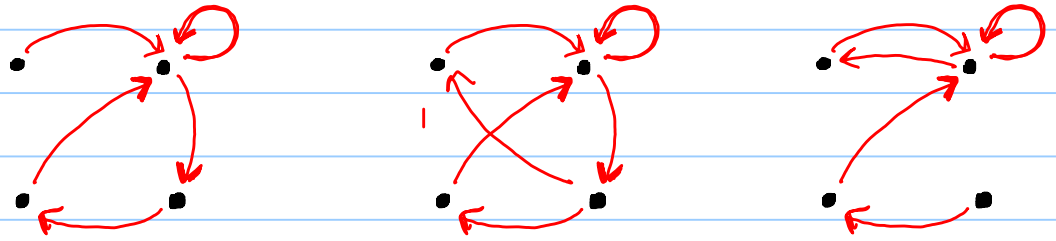
Trans

A relation R on set S is transitive if

$$xRy \text{ and } yRz \implies xRz.$$

Never have:





P on $\mathbb{Z}$ $\quad xPy$ iff $\exists\ n \in \mathbb{N}$ $\quad x^n = y$.

M $\quad sMt$ if s & t have the same birthday.

E $\quad sEt$ if s earns more money than t.