Note Title 3/11/2015

Partial Order

- reflexive.
- anti-symmetric
- transitive

Strict Order

- anti-refl
- anti-symm.
- transitive.

Total Order: if $x \neq y \Rightarrow$ then either xRy or yRx.

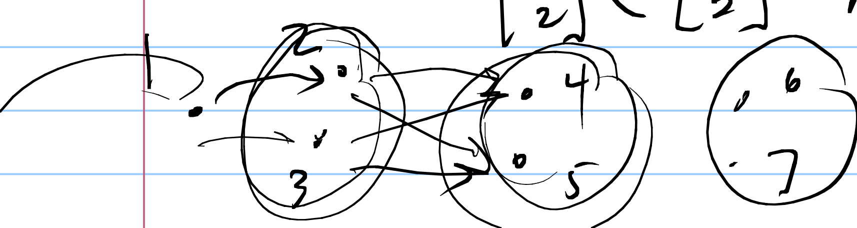
$$xRy \iff \left\lfloor \frac{x}{2} \right\rfloor < \left\lfloor \frac{y}{2} \right\rfloor$$

Reflexive xRx? $\left\lfloor \frac{x}{2} \right\rfloor < \left\lfloor \frac{x}{2} \right\rfloor$? never possible

$\Rightarrow$ anti-reflexive

Anti-Symmetric $xRy \wedge yRx \Rightarrow x=y$.

$$\left\lfloor \frac{x}{2} \right\rfloor < \left\lfloor \frac{y}{2} \right\rfloor \wedge \left\lfloor \frac{y}{2} \right\rfloor < \left\lfloor \frac{x}{2} \right\rfloor \Rightarrow x=y.$$



$x = 1 \quad \left\lfloor \frac{x}{2} \right\rfloor = 0 \qquad \left\lfloor \frac{4}{2} \right\rfloor = 2$

$2 \quad \left\lfloor \frac{2}{2} \right\rfloor = 1$

$3 \quad \left\lfloor \frac{3}{2} \right\rfloor = 1. \qquad \left\lfloor \frac{5}{2} \right\rfloor = 2.$

Def of Trans:

$$\overbrace{\left\lfloor \frac{x}{2} \right\rfloor < \left\lfloor \frac{y}{2} \right\rfloor}^{xRy} \ \& \ \overbrace{\left\lfloor \frac{y}{2} \right\rfloor < \left\lfloor \frac{z}{2} \right\rfloor}^{yRz} \Rightarrow$$

$$\left\lfloor \frac{x}{2} \right\rfloor < \left\lfloor \frac{z}{2} \right\rfloor \Rightarrow xRz.$$

$$\left\lfloor \frac{x}{2} \right\rfloor \leq \left\lfloor \frac{y}{2} \right\rfloor$$

$x = 3$ $\quad 3R2$

$y = 2$ $\quad 2R3$

$$\overset{1}{\left\lfloor \frac{3}{2} \right\rfloor} \leq \overset{1}{\left\lfloor \frac{2}{2} \right\rfloor}$$

$$\underset{1}{\left\lfloor \frac{2}{2} \right\rfloor} \leq \underset{1}{\left\lfloor \frac{3}{2} \right\rfloor}$$

$$\begin{tikzcd} 2 \arrow[r, bend left] & 3 \arrow[l, bend left] \end{tikzcd}$$

$\mathbb{Z}\times\mathbb{Z}$ $\qquad (a,b)\ R\ (c,d)$

iff $\quad a<c$ and $b<d$.

1. $(a,b)\ R\ (a,b)$? $\qquad$ Anti-reflexive

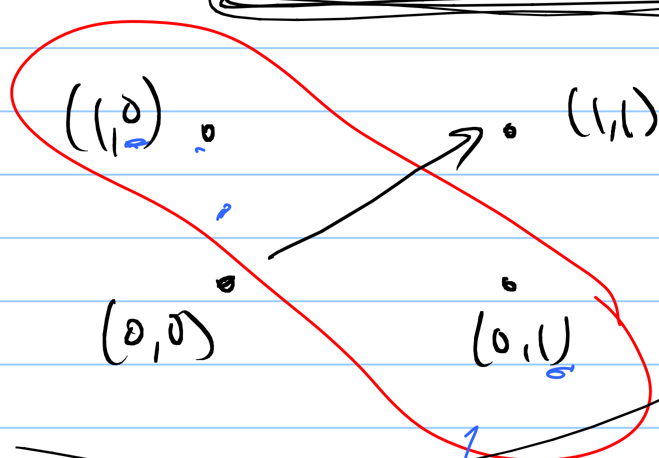
$a<a$ & $b<b$.

2. $(a,b)\ R\ (c,d)$ $\qquad$ then $(a,b)=(c,d)$

$(c,d)\ R\ (a,b)$

if $\boxed{\begin{array}{l} a<c \text{ and } b<d \\ c<a \text{ and } d<b \end{array}}$ then $\quad a=c$, $b=d$.

anti-symmetric



$1<0 \wedge 0<1$.

$0<1 \wedge 1<0$.

not comparable.

Strict order & Total Order

if $x \neq y$ then xRy or yRx

$$\overline{x+y} = \bar{x}\bar{y} \longleftrightarrow x+y = \overline{\bar{x}\bar{y}}$$

$$\overline{xy} = \bar{x}+\bar{y} \longleftrightarrow xy = \overline{\bar{x}+\bar{y}}$$

$$x \uparrow y = \overline{xy} \qquad \overline{x \uparrow y} = xy$$

$$x \downarrow y = \overline{x+y} \qquad \overline{x \downarrow y} = x+y$$

$$x \uparrow x = x \downarrow x = \bar{x}$$

$$a+b = \overline{\bar{a}\bar{b}} = \bar{a} \uparrow \bar{b}$$

$$(a \uparrow a) \uparrow (b \uparrow b)$$

$$ab = \overline{\bar{a}+\bar{b}} \qquad \bar{a} \downarrow \bar{b} = (a \downarrow a) \downarrow (b \downarrow b)$$

$$\underbrace{x}_{a}\underbrace{y}_{bc} = \overline{x \uparrow y} \qquad \bar{x} = x \uparrow x$$

$$\underbrace{a}\underbrace{bc}$$

$$= \overline{a \uparrow bc} = \overline{a \uparrow (\overline{b \uparrow c})}$$

$$= \overline{a \uparrow (b \uparrow c) \uparrow (b \uparrow c)}$$

$$= (a \uparrow ((b \uparrow c) \uparrow (b \uparrow c))) \uparrow (\qquad)$$

Direct Proof
Proof by Contrapositive] for proving $h \to c$.

Proof by contradiction.

Proof by cases.

Direct: Assume h & show c follows.

Contrapos: Assume $\neg c$ and show $\neg h$ follows.

Theorem: $(h_1 \wedge h_2 \wedge h_3) \to c$.

Sufficient to show $(h_1 \wedge h_2 \wedge \neg c) \to \neg h_3$

Theorem If n is an integer & n^2 is even then n is even.

[Assume n is an int & n is not even then n^2 is not even.] contra pos.

$n = 2k+1$ $\quad$ k some int.

$\hookrightarrow \quad n^2 = (2k+1)^2$

$n^2 = 2k$