Ordered Items

$(\_ , \_)$ notation used to denote an ordered pair of items.

For example: $(1,2) \neq (2,1)$

Whereas: $\{1,2\} = \{2,1\}$

Can also have ordered triplets:

Ex: $(2,1,3) \neq (3,1,2)$

Or ordered n-tuples:

$$\underbrace{(5, 1, -2, \ldots, 17)}_{n \text{ ordered items.}}$$

Cartesian Products

Given sets A & B, the Cartesian product of A & B (denoted $A \times B$) is the set of all pairs (a, b) where $a \in A$ and $b \in B$.

Example: $A = \{1, 2\}$ $B = \{x, y, z\}$

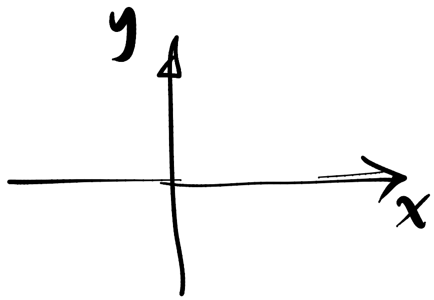
$$A \times B = \{(1,x), (1,y), (1,z), (2,x), (2,y), (2,z)\}$$

$(1,y) \in A \times B$.

Cartesian Product: More Examples

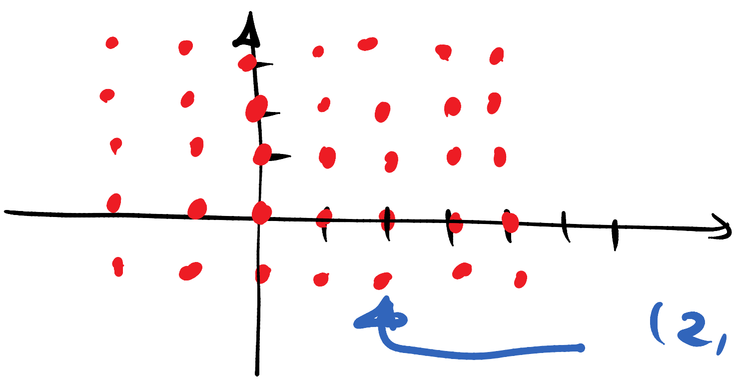
$$\mathbb{R} \times \mathbb{R} = \{(x,y) : x \in \mathbb{R}, y \in \mathbb{R}\}$$

all points in the plane.



$$\mathbb{Z} \times \mathbb{Z} = \{(x,y) : x \in \mathbb{Z} \text{ and } y \in \mathbb{Z}\}$$

an infinite grid of points.



(2,−1). for example.

if $A \neq B$ then $A \times B \neq B \times A$:

For example: $A = \{1, 2\}$

$B = \{x, y, z\}$

$(1, x) \in A \times B$ $(x, 1) \notin A \times B$

$(x, 1) \notin B \times A$ $(x, 1) \in B \times A$.

$A \times A$ is sometimes denoted A^2

Example: $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$.

Cartesian Products on Many Sets

$A = \{1, 2\}$

$B = \{x, y, z\}$

$C = \{a, b\}$

$A \times B \times C$ is the set of all triplets:

$$(\underbrace{\_\_}_{\text{element of } A}, \underbrace{\_\_}_{\text{element of } B}, \underbrace{\_\_}_{\text{element of } C.})$$

$$(1, y, b) \in A \times B \times C$$

$$(a, z, 2) \in C \times B \times A$$

If you have n sets: A_1 A_2 $\cdots$ A_n
then

$A_1 \times A_2 \times \cdots \times A_n = \{ (a_1, a_2, \ldots, a_n) :$
where $a_1 \in A_1$, $a_2 \in A_2, \ldots,$ $a_n \in A_n$.

$$A \times A \times A = A^3$$

$$A \times A \times A \times A = A^4$$

$$\vdots$$

$$\underbrace{A \times A \times \cdots \times A}_{n \text{ times}} = A^n$$

Strings

If A is a set of symbols elements in A^n can be denoted by strings without the parens and commas.

For example: $A = \{a, b\}$

an element of A^3 can be written as bba instead of (b, b, a).

Sets of Strings

$\{0,1\}^3 = \{000, 001, 010, 011, 100, 101, 110, 111\}$

= set of all binary strings with 3 bits.

$D = \{0,1,2,3,4,5,6,7,8,9\}$

$A = \{a, b, c, \ldots, z\}$

$5zbg \in D \times A \times A \times A. = D \times A^3$

Concatenation

Strings $s = 011$

$t = 1110$

The concatenation of s & t (denoted st) is 0111110

If $x \in \{0,1\}^3$

$y \in \{0,1\}^5$

then $xy \in \{0,1\}^8$

The empty string has no characters and is denoted by λ.

$$x\lambda = \lambda x = x.$$

$$A^0 = \{\lambda\}.$$

Recap of Proofs by Contradiction.

Theorem: $\sqrt{2}$ is an irrational number.

The proof of the theorem makes use of the following fact:

If n is an integer and n^2 is even then n is also even.

Proof Overview

Assume $\sqrt{2}$ is rational.

$\sqrt{2} = \frac{n}{d}$ for some integers n + d where n+d do not have a common divisor. (Reduced form).

$\Downarrow$

$2 = \frac{n^2}{d^2}$

$\Rightarrow$ d and n are both even. (2 divides n + 2 divides d).

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CONTRADICTION!

$\therefore \sqrt{2}$ is irrational

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$2k^2 = d^2$ divide by 2.

d^2 is even $\Rightarrow$ d is even.