Note Title 2/10/2015

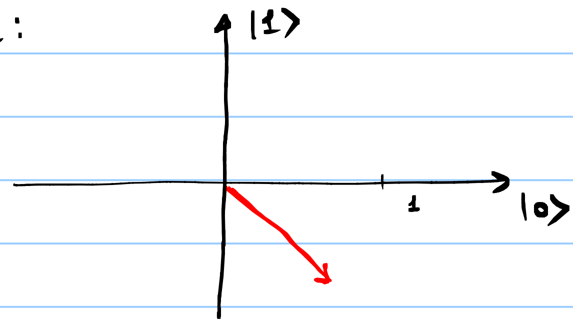
Problem Set : Quantum Computation

University Studies 3

Due in class on Monday March 2.

In the quantum state $\alpha|0\rangle + \beta|1\rangle$, the amplitudes α and β, can be complex numbers. However, to keep things simpler, we will stick with the assumption we have made in class that the amplitudes will be real numbers.

1) In class, we plotted quantum bit states on a 2-dimensional plot. For example, the state $\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$ would be:



a) We will define the state $|\phi\rangle$ to be: $|\phi\rangle = -\frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle$.
Plot $|\phi\rangle$.

b) What are the two states that are orthogonal to $|\phi\rangle$?

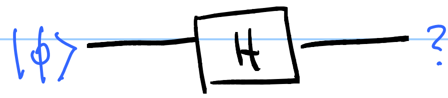
c) Plot the two states you got for part b).

d) If you were to measure the state $|\phi\rangle$ to determine if it is $|0\rangle$ or $|1\rangle$, then what is the probability of each outcome?

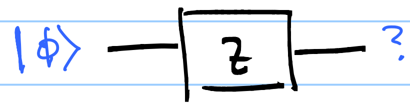
e) Find α and β such that $\alpha|+\rangle + \beta|-\rangle = |\phi\rangle$.

e) If you were to measure the state $|\phi\rangle$ to determine if it is $|+\rangle$ or $|-\rangle$, then what is the probability of each outcome?

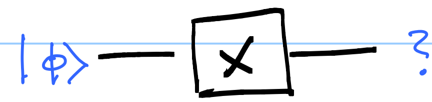
f) What is the result of applying the Haddamard gate to $|\phi\rangle$?



g) What is the output of:



h) What is the output of



2. Imagine measuring the first qubit in each of the following 2-qubit states. What is the probability of getting a $|0\rangle$ or a $|1\rangle$? What is the resulting state for each outcome?

a) $\frac{1}{\sqrt{3}}|01\rangle + \sqrt{\frac{2}{3}}|10\rangle$

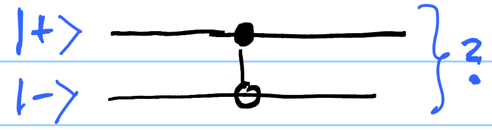
b) $\frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle$

c) $\frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|00\rangle$

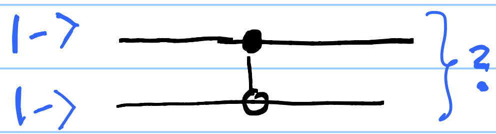
d) $\frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|11\rangle$

3. Determine the outputs of the following circuits. In each case, you should be able to "factor" the output state into two separate 1-qubit states.

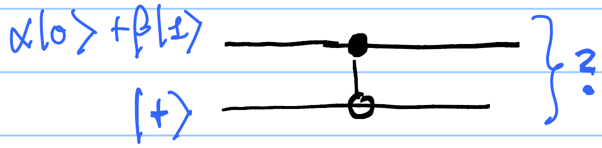
a)



b)



c)



d)