Quantum Teleportation.

Note Title 2/3/2015

Two parties, Alice & Bob. Alice has a qubit in an unknown state. She wants to transmit the state to Bob without measuring or destroying the state.

She will have to send two classical bits to Bob. Also she and Bob will have to share an entangled pair of qubits.

Alice's qubit is $a|0\rangle + b|1\rangle$
a & b can be high precison numbers and they are communicated to Bob by only transmitting 2 classical bits of information.

Here is the state of the system at the beginning:

Alice	Alice	Bob.
$a|0\rangle + b|1\rangle$	$\frac{1}{\sqrt{2}}|0$ - - - — — - -	$0\rangle$
Q1	$\frac{1}{\sqrt{2}}|1$ - - -	$1\rangle$
	Q2	Q3

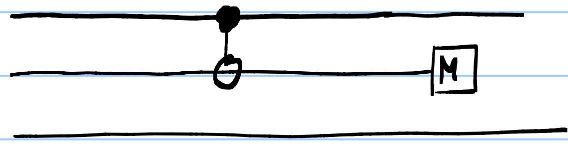
We will depict our circuit as:

1 ______ Alice ______

2 ______ Alice ______

3 ______ Bob ______

$$\left(a|0\rangle + b|1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) = \begin{matrix} \frac{a}{\sqrt{2}}|000\rangle \\ \frac{a}{\sqrt{2}}|011\rangle \\ b/\sqrt{2}\,|100\rangle \\ b/\sqrt{2}\,|111\rangle \end{matrix}$$



CNOT
Q1: control
Q2: target

$$\frac{a}{\sqrt{2}}\left(|000\rangle + |011\rangle\right) + \frac{b}{\sqrt{2}}\left(|100\rangle + |111\rangle\right)$$

$$\frac{a}{\sqrt{2}}\left(|000\rangle + |011\rangle\right) + b/\sqrt{2}\left(|110\rangle + |101\rangle\right)$$

Alice measures middle qubit.

0: $a|000\rangle + b|101\rangle$

1: $a|011\rangle + b|110\rangle$

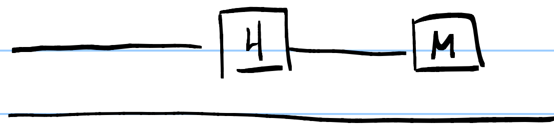
Alice sends the bit she measures to Bob.
If 0 then Bob does nothing.
If 1 then Bob flips his qubit:

$a|000\rangle + b|101\rangle$ $\qquad$ $a|010\rangle + b|111\rangle$

Middle qubit is the outcome of the measurement
So we will ignore it:

State $\quad a|00\rangle + b|11\rangle$

Alice has qubit 1
Bob has qubit 2.



$a|00\rangle + b|11\rangle$

$a \cdot \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle \qquad b \cdot \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)|1\rangle$

$$\frac{a}{\sqrt{2}}|00\rangle + \frac{b}{\sqrt{2}}|01\rangle + \frac{a}{\sqrt{2}}|10\rangle - \frac{b}{\sqrt{2}}|11\rangle$$

Measure Qubit 1

$\xrightarrow{0}$ $a|00\rangle + b|01\rangle \qquad \xrightarrow{1}$ $a|10\rangle - b|11\rangle$

$|0\rangle \otimes (a|0\rangle + b|1\rangle) \qquad |1\rangle \otimes (a|0\rangle - b|1\rangle)$

Alice sends the result of her measurement to Bob.
If it's 0, Bob does nothing.
If it's 1, Bob performs a "Z" operation.

$|0\rangle \otimes (a|0\rangle + b|1\rangle) \qquad |1\rangle \otimes (a|0\rangle + b|1\rangle)$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \begin{aligned} |0\rangle &\longrightarrow |0\rangle \\ |1\rangle &\longrightarrow -|1\rangle. \end{aligned}$$

Note

The state $a|00\rangle + b|01\rangle$ can be written so that qubit 1 is in a separate state than qubit 2:

$$|0\rangle \otimes (a|0\rangle + b|1\rangle)$$