Quantum Teleportation.

Two parties, Alice and Bob. Alice has a qubit in an unknown state. She wants to transmit the state to Bob without measuring or destroying the state.

She will have to send two classical bits to Bob. Also, she and Bob will have to share an entangled pair of qubits.

Alice's qubit is \( a|0\rangle + b|1\rangle \).

\( a \) and \( b \) can be high precision numbers and they are communicated to Bob by only transmitting 2 classical bits of information.

Here is the state of the system at the beginning:

\[ \begin{array}{cccc}
\text{Alice} & \text{Alice} & \text{Bob} \\
|0\rangle & |1\rangle & \frac{1}{\sqrt{2}} |00\rangle & |\pm\rangle & \frac{1}{\sqrt{2}} |01\rangle & |0\rangle \\
Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\
\end{array} \]

We will depict our circuit as:

1. Alice
2. Alice
3. Bob

\( (a|0\rangle + b|1\rangle) \otimes \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) = \frac{a}{\sqrt{2}} |1000\rangle + \frac{b}{\sqrt{2}} |1011\rangle \)
\[ \frac{a}{\sqrt{2}} \left( |000\rangle + |111\rangle \right) + \frac{b}{\sqrt{2}} \left( |100\rangle + |111\rangle \right) \]

Alice measures middle qubit.

Alice sends the bit. She measures to Bob.

If 0 then Bob does nothing.
If 1 then Bob flips his qubit:

\[ a |000\rangle + b |101\rangle \quad \text{and} \quad a |010\rangle + b |111\rangle \]

Middle qubit is the outcome of the measurement, so we will ignore it:

State \[ a |00\rangle + b |11\rangle \]

Alice has qubit 1.
Bob has qubit 2.
\( a |00\rangle + b |11\rangle \)

\[
a \cdot \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) |0\rangle \\
b \cdot \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) |1\rangle
\]

\[
\frac{a}{\sqrt{2}} |00\rangle + \frac{b}{\sqrt{2}} |01\rangle + \frac{a}{\sqrt{2}} |10\rangle - \frac{b}{\sqrt{2}} |11\rangle
\]

Measure Qubit 0

\[
a |00\rangle + b |01\rangle \\
a |10\rangle - b |11\rangle
\]

\[
|0\rangle \oplus (a|0\rangle + b|1\rangle) \\
|1\rangle \oplus (a|0\rangle - b|1\rangle)
\]

Alice sends the result of her measurement to Bob.
If it's 0, Bob does nothing.
If it's 1, Bob performs a "\(\frac{1}{2}\)" operation.

\[
Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \\
|0\rangle \rightarrow |0\rangle \\
|1\rangle \rightarrow -|1\rangle
\]
The state $a|00\rangle + b|01\rangle$ can be written so that qubit 1 is in a separate state than qubit 2:

$|0\rangle \otimes (a|0\rangle + b|1\rangle)$