

STATISTICS 110/201 PRACTICE FINAL EXAM KEY (REGRESSION ONLY)

**Questions 1 to 5:** The last page of this exam gives output for the following situation. The data consist of the 68 houses from Appendix C7 that have Quality = 1. Y = Sales Price of the house (“salesprice” on the output), and the three predictor variables are:

$X_1$  = Square Feet divided by 100 = “sqft100” on the output

$X_2$  = Number of bedrooms = “bedrooms” on the output

$X_3$  = Lot Size in square feet = “lotsize” on the output

The model used did not involve any transformations; it is  $E\{Y_i\} = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3}$ .

The output is from a package called Stata, but all packages will do these analyses. The model was fit using Type III Sums of Squares (“Adjusted”) and then repeated using Sequential Sums of Squares (Type I). In other words, the SS is built up as each variable is added, in the order they are given in the command.

Notice in the output that the model was fit twice, with the variables in two different orders, but we will keep the designation of  $X_1$ , etc as defined above. In other words, we define  $X_1$  = Square Feet divided by 100, and so on, no matter what order they appear in the computer command.

NOTE: In case you aren’t familiar with this notation,  $1.18e+11 = 1.18 \times 10^{11}$ , as an example.

1. Write the estimated regression equation for the full model with all 3 variables, filling in numbers for the coefficients.

$$\hat{Y} = 290558.1 + 9893.524X_1 - 36018.68X_2 + 2.513X_3$$

2. Carry out a test of the null hypothesis  $H_0: \beta_1 = \beta_2 = \beta_3 = 0$ . State the test statistic, the  $p$ -value, and your conclusion about whether or not to reject the null hypothesis using  $\alpha = .05$ .

*Test statistic:  $F = 6.69$*

*$p$ -value = 0.0005*

*Reject the null hypothesis (because  $0.0005 < 0.05$ )*

3. Give numerical values for each of the following, where  $X_1$ ,  $X_2$  and  $X_3$  are defined above:

a.  $SSR(X_2) = 4.02 \times 10^7$  (from the “bedrooms” row and Seq.SS column for the 2<sup>nd</sup> version)

b.  $SSR(X_3 | X_1, X_2) = 6.19 \times 10^{10}$  (from the “lotsize” row for either version)

c.  $SSR(X_1 | X_2) = 1.87 \times 10^{11}$  (find this in the “sqft100” row for the 2<sup>nd</sup> version)

d. The  $p$ -value for testing  $H_0: \beta_3 = 0$ , given that  $X_1$  and  $X_2$  are in the model = 0.0291

e. The  $p$ -value for testing  $H_0: \beta_2 = 0$ , given that the other  $X$ ’s are *not* in the model = 0.9548

(note that the  $F^*$  value is only .003)

4. Give the numerical values (from the output) for two different test statistics for testing  $H_0: \beta_3 = 0$  (given that  $X_1$  and  $X_2$  are in the model). Then explain in words, in the context of the real estate situation, what this hypothesis is testing.

$$t = 2.23$$

$$F = 4.989$$

*(The above two test statistics are equivalent and give a corresponding p-value of 0.029)*

*The hypothesis is testing whether the predictor variable Lot Size needs to be in the model, given that square feet and number of bedrooms are already in the model. Or you could say that it tests whether Lot Size is a statistically significant predictor variable, given that square feet and number of bedrooms are in the model.*

5. If we were to fit the simple linear regression model using bedrooms as the only predictor, would the result be that number of bedrooms is a significant predictor of Sales Price? Explain how you know, using information provided in the output.

*No, number of bedrooms by itself is not a significant predictor. The appropriate test is given in the "bedrooms" row of the last table. The test statistic is  $F = 0.0032$  and the p-value is 0.9548, which is clearly much larger than 0.05.*

-----END OF QUESTIONS BASED ON THE COMPUTER OUTPUT-----

6. When examining case diagnostics in multiple regression, under what circumstance is it acceptable to remove a case that is clearly a Y outlier?

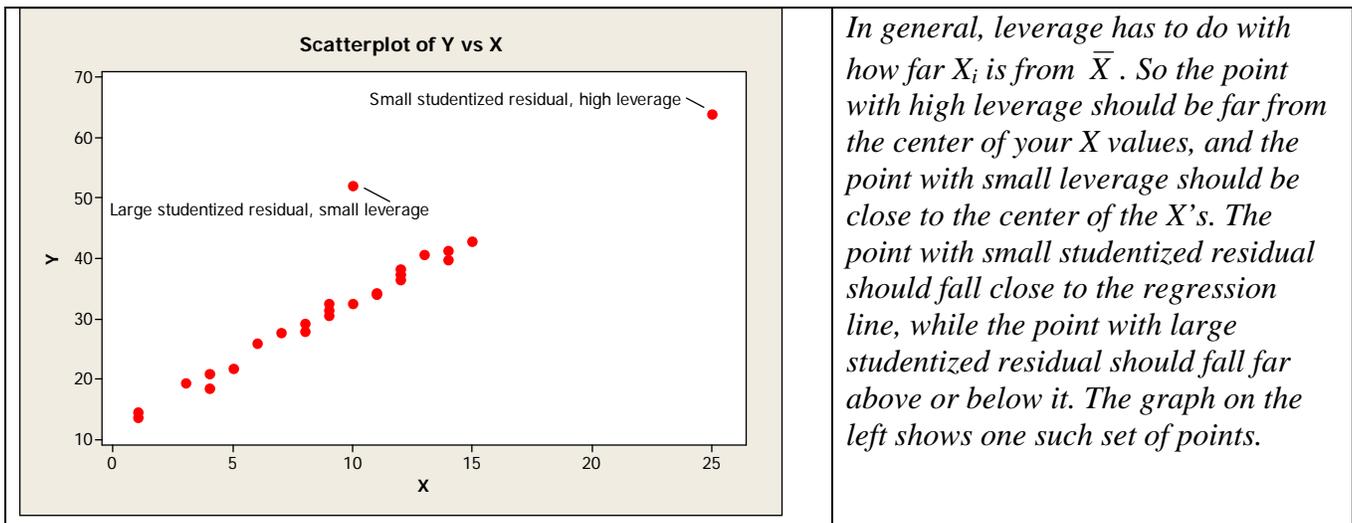
*It is only acceptable to remove a Y outlier if it is clearly a mistake.*

7. Give two circumstances in which it is acceptable to remove one or more cases that are outliers in the X variables.

1. *One or more of the X values for the case is clearly a mistake.*

2. *If several cases have X values different from the rest of the data, and the prediction does not work well for cases with those combinations of X value, then it's likely that the cases belong to a different population and the relationship between Y and the predictor variables is not the same as it is for the main population. The cases can be removed, but the model would not apply in the future for predicting Y when X has values in that area.*

8. Draw a scatter plot of Y versus X showing points for a simple linear regression analysis, illustrating a case that has a small studentized residual but high leverage, and a case that has a large studentized residual but small leverage. Make sure you label which point is which.



*In general, leverage has to do with how far  $X_i$  is from  $\bar{X}$ . So the point with high leverage should be far from the center of your X values, and the point with small leverage should be close to the center of the X's. The point with small studentized residual should fall close to the regression line, while the point with large studentized residual should fall far above or below it. The graph on the left shows one such set of points.*

9. Suppose you have four possible predictor variables ( $X_1$ ,  $X_2$ ,  $X_3$ , and  $X_4$ ) that could be used in a regression analysis. You run a forward selection procedure, and the variables are entered as follows: Step 1:  $X_2$     Step 2:  $X_4$     Step 3:  $X_1$     Step 4:  $X_3$

In other words, after Step 1, the model is  $E\{Y\} = \beta_0 + \beta_1 X_2$

After Step 2, the model is  $E\{Y\} = \beta_0 + \beta_1 X_2 + \beta_2 X_4$

And so on...

You also run an all subsets regression analysis using  $R^2$  as the criterion for the "best" model for each possible number of predictors (1, 2, 3, 4). Would the same models result from this analysis as from the forward stepwise procedure? In other words, would "all subsets regression" definitely identify the following as the best models for 1, 2, 3, and 4 variables? Circle Yes or No in each case.

a.  $\beta_0 + 1$  variable, best model would be  $E\{Y\} = \beta_0 + \beta_1 X_2$  **YES**

b.  $\beta_0 + 2$  variables, best model would be  $E\{Y\} = \beta_0 + \beta_1 X_2 + \beta_2 X_4$  **NO**

c.  $\beta_0 + 3$  variables, best model would be  $E\{Y\} = \beta_0 + \beta_1 X_2 + \beta_2 X_4 + \beta_3 X_1$  **NO**

d.  $\beta_0 + 4$  variables, best model would be  $E\{Y\} = \beta_0 + \beta_1X_2 + \beta_2X_4 + \beta_3X_1 + \beta_4X_3$       **YES**

10. An international company is worried that employees in a certain job at its headquarters in Country A are not being given raises at the same rate as employees in the same job at its headquarters in Country B. Using a random sample of employees from each country, a regression model is fit with:
- $Y$  = employee salary  
 $X_1$  = length of time employee has worked for the company  
 $X_2$  = 1 if employee is in Country A, and 0 if employee is in Country B.

New employees, who have  $X_1 = 0$ , all start at the same salary, so the company is not interested in fitting a model with different intercepts, only with different slopes.

- a. Write the full and reduced models for determining whether or not the slopes are different for employees in the two countries, using the variable definitions above and standard notation.

Full model  $E\{Y\} = \beta_0 + \beta_1X_1 + \beta_2X_1X_2$

Reduced model:  $E\{Y\} = \beta_0 + \beta_1X_1$

- b. For the full model, write the row of the X matrix for an employee with 10 years of experience in Country A, and the row of the X matrix for an employee with 12 years of experience in Country B. You should write these using numbers, not symbols.

*For 10 years experience in Country A: [1 10 10]*

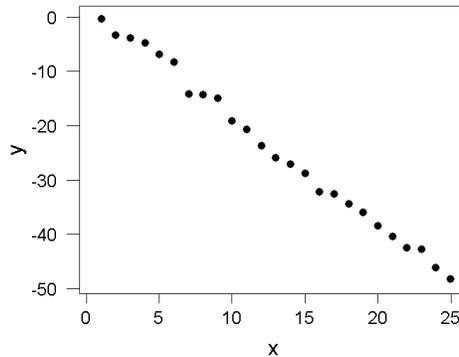
*For 12 years experience in Country B: [1 12 0]*

## **MULTIPLE CHOICE QUESTIONS**

**Circle the best answer.**

1. In a linear regression analysis with the usual assumptions (stated on page 218 and other places in the text), which one of the following quantities is the same for all individual units in the analysis?
- A. Leverage  $h_{ii}$   
**B.  $s\{Y_i\}$**   
 C.  $s\{e_i\}$   
 D.  $s\{\hat{Y}_i\}$
2. A regression line is used for all of the following *except* one. Which one is *not* a valid use of a regression line?
- A. to estimate the average value of Y at a specified value of X.  
 B. to predict the value of Y for an individual, given that individual's X-value.  
 C. to estimate the change in Y for a one-unit change in X.  
**D. to determine if a change in X causes a change in Y.**

3. Which choice is *not* an appropriate description of  $\hat{Y}$  in a regression equation?
- Estimated response
  - Predicted response
  - Estimated average response
  - Observed response**
4. Which of the following is the *best* way to determine whether or not there is a statistically significant linear relationship between two quantitative variables?
- Compute a regression line from a sample and see if the sample slope is 0.
  - Compute the correlation coefficient and see if it is greater than 0.5 or less than  $-0.5$ .
  - Conduct a test of the null hypothesis that the population slope is 0.**
  - Conduct a test of the null hypothesis that the population intercept is 0.
5. Shown below is a scatterplot of Y versus X.



Which choice is most likely to be the approximate value of  $R^2$ ?

- $-99.5\%$
  - $2.0\%$
  - $50.0\%$
  - $99.5\%$**
6. In a regression model with  $p - 1$  predictor variables chosen from a set of  $P - 1$  possible predictor variables, which of the following indicates that bias is *not* a problem with the model?
- Mallow's  $C_p \leq p$  (small p)**
  - Mallow's  $C_p \leq P$  (cap P)
  - Mallow's  $C_p > p$  (small p)
  - Mallow's  $C_p > P$  (cap P)
7. Which of the following case diagnostic measures is based on Y values only (and not X values)?
- Cook's Distance
  - Studentized deleted residual
  - DFFITs
  - None of the above – they all use the X values and the Y values**
8. Which of the following methods is the most appropriate for testing  $H_0: \beta_k = 0$  versus  $H_a: \beta_k > 0$ ?
- A t-test**
  - An F-test
  - A test of a full versus reduced model
  - All of the above are equally good.

9. Which of the following is *not* a valid null hypothesis?
- A.  $H_0: \beta_1 = 0$
  - B.  $H_0: \beta_1 = \beta_2$
  - C.  $H_0: \mathbf{b}_1 = \mathbf{b}_2 = \mathbf{0}$**
  - D. All of the above *are* valid null hypotheses
10. Which of the following can never be 0 (unless the population standard deviation  $\sigma = 0$ )?
- A. The estimated intercept,  $b_0$
  - B. A studentized deleted residual,  $t_i$
  - C. The variance of the prediction error,  $\sigma^2\{\text{pred}\}$**
  - D. The estimate of  $E\{Y_h\}$ ,  $\hat{Y}_h$

Here is the Computer Output for Questions 1 to 5.

**Model salesprice using sqft bedrooms lotsize using “Adjusted” sum of squares**

Source	SS	df	MS	Number of obs =	67
Model	2.4910e+11	3	8.3035e+10	F( 3, 63) =	6.69
Residual	7.8158e+11	63	1.2406e+10	Prob > F =	0.0005
				R-squared =	0.2417
				Adj R-squared =	0.2056
Total	1.0307e+12	66	1.5616e+10	Root MSE =	1.1e+05

salesprice	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
sqft100	9893.524	2508.561	3.94	0.000	4880.564	14906.48
bedrooms	-36018.68	14665.17	-2.46	0.017	-65324.68	-6712.675
lotsize	2.512511	1.124531	2.23	0.029	.2653148	4.759707
_cons	290558.1	75398.79	3.85	0.000	139885.6	441230.5

**Same model as above, using “Sequential” sum of squares:**

salesprice	Coef.	Seq SS	df1	df2	F	Prob > F
sqft100	6187.993	1.18e+11	1	63	9.474134	0.0031
bedrooms	-34701.03	6.96e+10	1	63	5.605086	0.0210
lotsize	2.512511	6.19e+10	1	63	4.989326	0.0291

**Model salesprice using bedrooms sqft lotsize (in that order) using “Adjusted” sum of squares**

Source	SS	df	MS	Number of obs =	67
Model	2.4910e+11	3	8.3035e+10	F( 3, 63) =	6.69
Residual	7.8158e+11	63	1.2406e+10	Prob > F =	0.0005
				R-squared =	0.2417
				Adj R-squared =	0.2056
Total	1.0307e+12	66	1.5616e+10	Root MSE =	1.1e+05

salesprice	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
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_cons	290558.1	75398.79	3.85	0.000	139885.6	441230.5

**Same model as above, using “Sequential” sum of squares (so bedrooms is in first, then sqft):**

salesprice	Coef.	Seq SS	df1	df2	F	Prob > F
bedrooms	-668.8945	4.02e+07	1	63	.0032419	0.9548
sqft100	9739.083	1.87e+11	1	63	15.07598	0.0003
lotsize	2.512511	6.19e+10	1	63	4.989326	0.0291