TODAY:

- Section 12.1, Lesson 3: What can go wrong with hypothesis testing
- Section 12.3: Hypothesis tests for difference in two proportions

ANNOUNCEMENTS:

- Make sure you check your grades on eee and notify me if any of them are incorrect or missing.
- Friday discussion is for credit this week. Last chance!

HOMEWORK (due Mon, March 11):

Chapter 12:

#62 and 64 (count together as 1, find p-values for #62), 90, 112

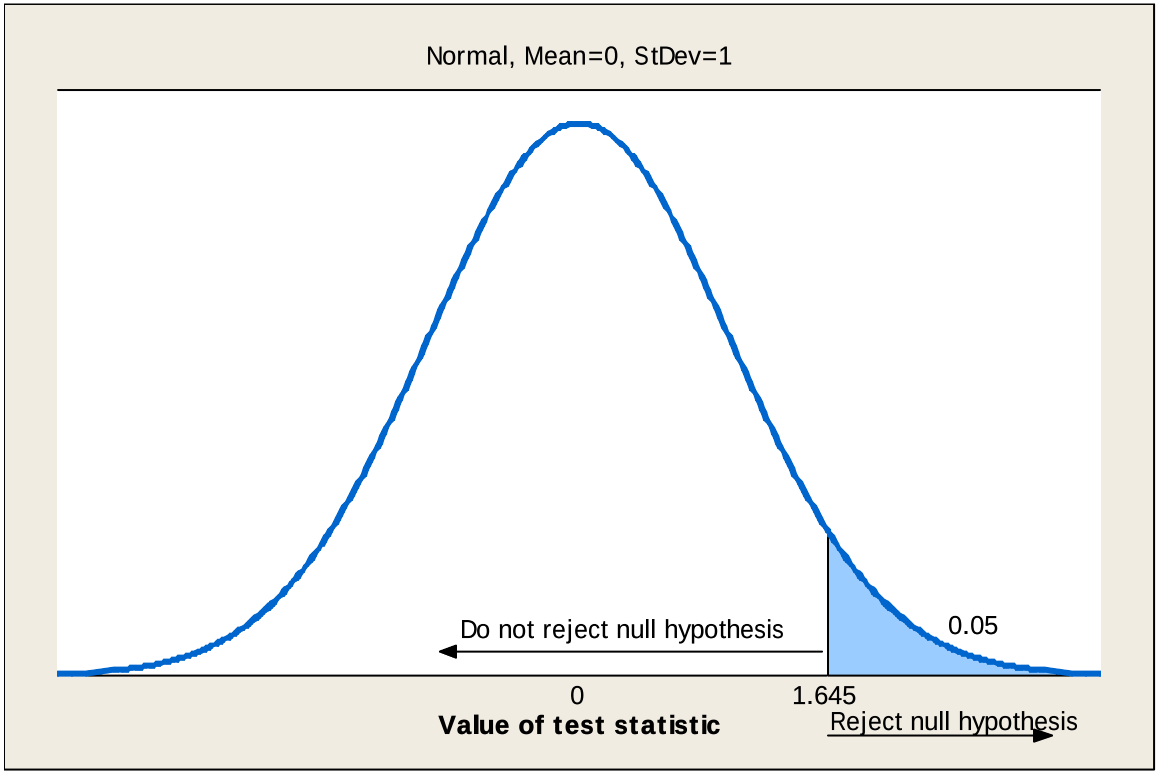
REVIEW OF HYPOTHESIS TEST FOR ONE PROPORTION
ONE-SIDED TEST, WITH PICTURE

H_0: $p = p_0$ versus Ha: $p > p_0$

Reject H_0 if *p*-value < .05. For what values of z does that happen?

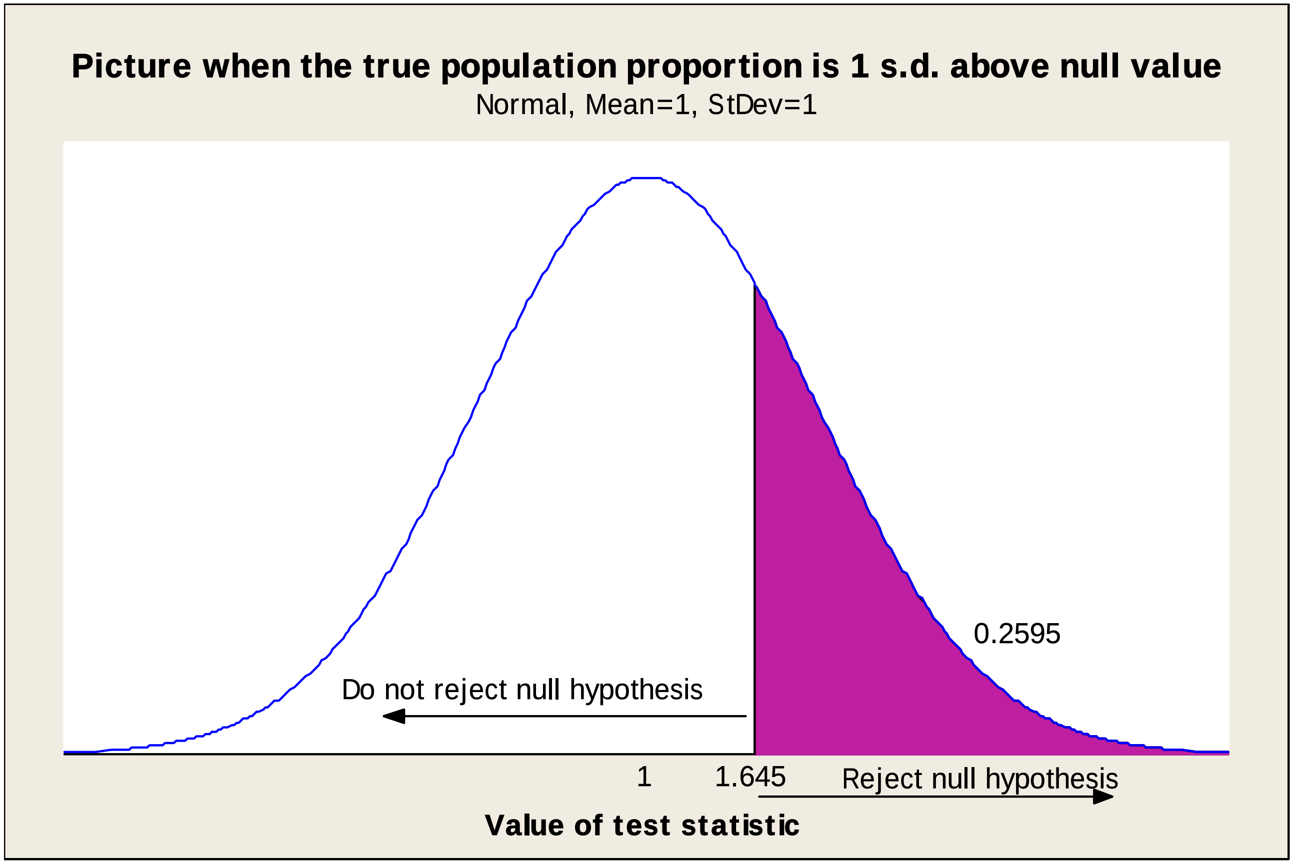
Notation: *level of significance* = α (alpha, usually .05)

p-value < .05 corresponds with z > 1.645 (rejection region)



AN ILLUSTRATION OF WHAT HAPPENS WHEN Ha IS TRUE

A Specific Example of finding *power: Suppose* the *truth* for the population proportion p is one standard deviation above the null value p_0. Then the mean for the standardized scores will be 1 instead of 0. How often would we (correctly) reject the null hypothesis in that case? Answer (purple region) is .2595 = "power"



Section 12.1, Lesson 3

What Can Go Wrong in Hypothesis Testing:
The Two Types of Errors and Their Probabilities

Type 1 error (*false positive*) occurs when:
- *Null hypothesis* is actually *true*, but
- Conclusion of test is to reject H_0 and *accept* H_a

Type 2 error (*false negative*) occurs when:
- *Alternative hypothesis* is actually *true*, but
- Conclusion is that we *cannot reject* H_0

Example: Case Study 1.6, aspirin and heart attacks. Found statistically significant relationship; *p-value* was $< .00001$.

Heart attack and aspirin example:

Null hypothesis: Proportion of men who would have heart attacks if taking aspirin (p_1) = Proportion of men who would have heart attacks if taking placebo (p_2).

In symbols: H_0: $p_1 = p_2$ or H_0: $p_1 - p_2 = 0$

Alternative hypothesis: The heart attack proportion is *lower* if men were to take aspirin than if they were not to take aspirin.

In symbols: H_a: $p_1 < p_2$ or H_0: $p_1 - p_2 < 0$

Type 1 error (*false positive*): Occurs if there really is *no relationship* between taking aspirin and heart attack prevention, but we conclude that there *is* a relationship.

Consequence: Good for aspirin companies! For consumers, possible side effects from aspirin, with no redeeming value.

Type 2 error (*false negative*):
Occurs if there *is* a relationship but the study *failed to find it*.

Consequence:
Miss out on recommending something that could save lives!

Which type of error is more serious?
Probably all agree that Type 2 is more serious.

Which could have occurred?
Type 1 error could have occurred.
Type 2 could not have occurred, because we *did* find a significant relationship.

Aspirin Example: Consequences of the decisions

Decision:

Truth:	**Don't reject H_0, Don't conclude aspirin works**	**Reject H_0, Conclude aspirin works**	**Which error could occur:**
H_0: Aspirin doesn't work	OK	Type 1 error: People take aspirin needlessly; may suffer side effects	Type 1
Ha: Aspirin works	Type 2 error: Aspirin could save lives but we don't recognize benefits	OK	Type 2
Which error could occur?	Type 2	Type 1	

Note that because H_0 *was rejected* in this study, we could only have made a Type 1 error, not a Type 2 error.

Some analogies to hypothesis testing:

Analogy 1: <u>Courtroom:</u>
Null hypothesis: Defendant is innocent.
Alternative hypothesis: Defendant is guilty

- The two possible conclusions are "not guilty" and "guilty."
- The conclusion "not guilty" is equivalent to "don't reject H_0."
- We don't say defendant is "innocent" just like we don't accept H_0 in hypothesis testing.

<u>Type 1 error</u> is when defendant is *innocent* but *gets convicted*

<u>Type 2 error</u> is when defendant is *guilty* but *does not get convicted*.

Which one is more serious??

Analogy 2: Medical test
Null hypothesis: You do not have the disease
Alternative hypothesis: You have the disease

Type 1 error: You *don't* have disease, but test says *you do*; a "false positive"

Type 2 error: You *do* have disease, but test says you do not; a "false negative"

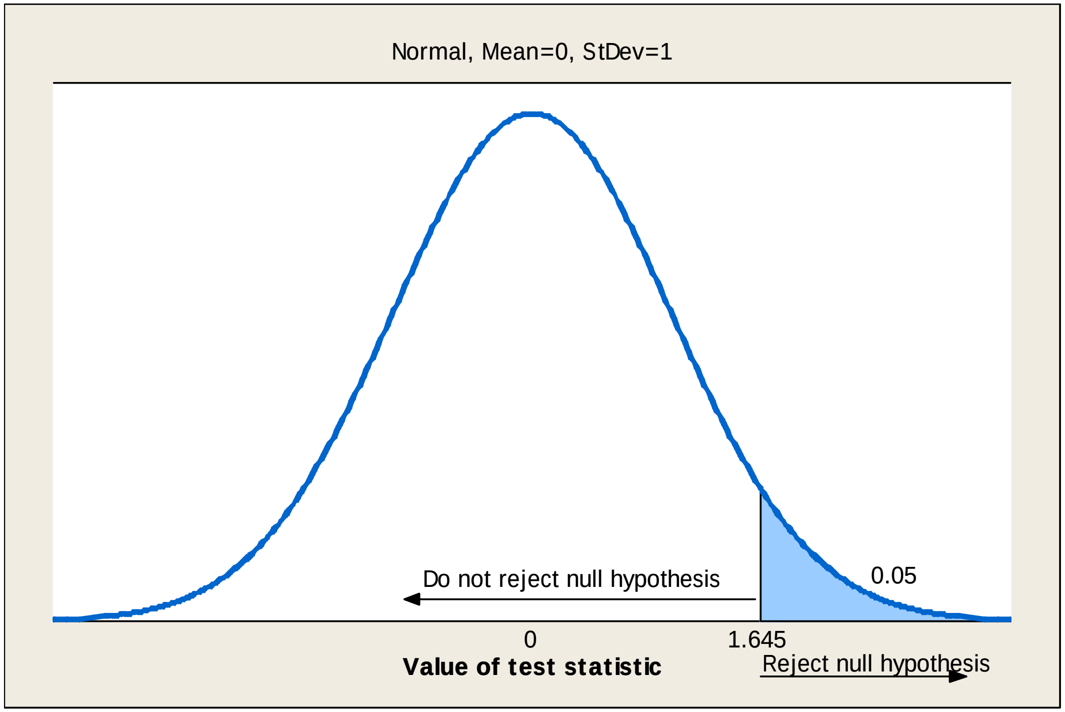
Which is more serious??

Notes and Definitions:

Probability related to Type 1 error:

The *conditional probability* of making a Type 1 error, given that H_0 is true, is the ***level of significance*** $\boldsymbol{\alpha}$. In most cases, this is .05. However, it should be adjusted to be lower (.01 is common) if a Type 1 error is *more serious* than a Type 2 error.

In probability notation: $P(\text{Reject } H_0 \mid H_0 \text{ is true}) = \alpha$, usually .05.

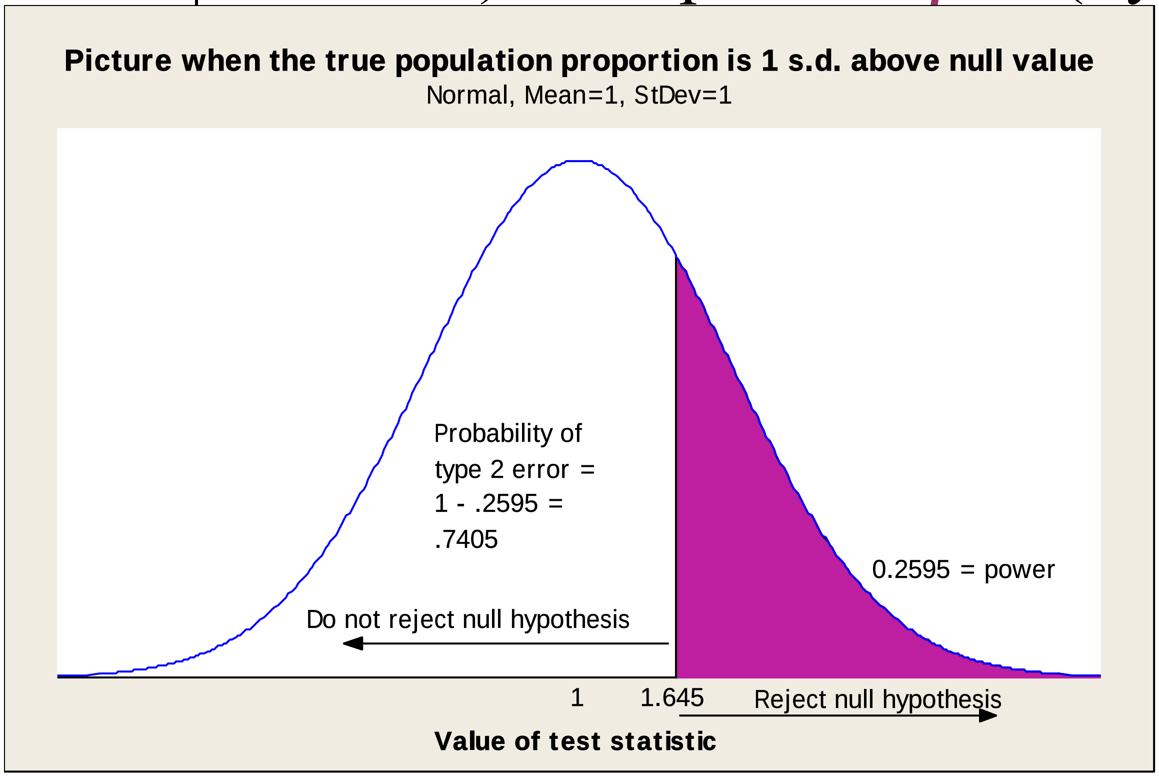


Probability related to Type 2 error and Power:

- *Conditional probability* of *correct decision*, given H_a is true is called the ***power*** of the test.
  - Can only calculate numerically for a *specific value* in H_a.
- Conditional probability of Type 2 error = 1 – power

P(Reject H_0 | H_a is true) = power.

P(Do not reject H_0 | H_a is true) = 1 – power = β =P(Type 2 error).



How can we increase power and decrease P(Type 2 error)?

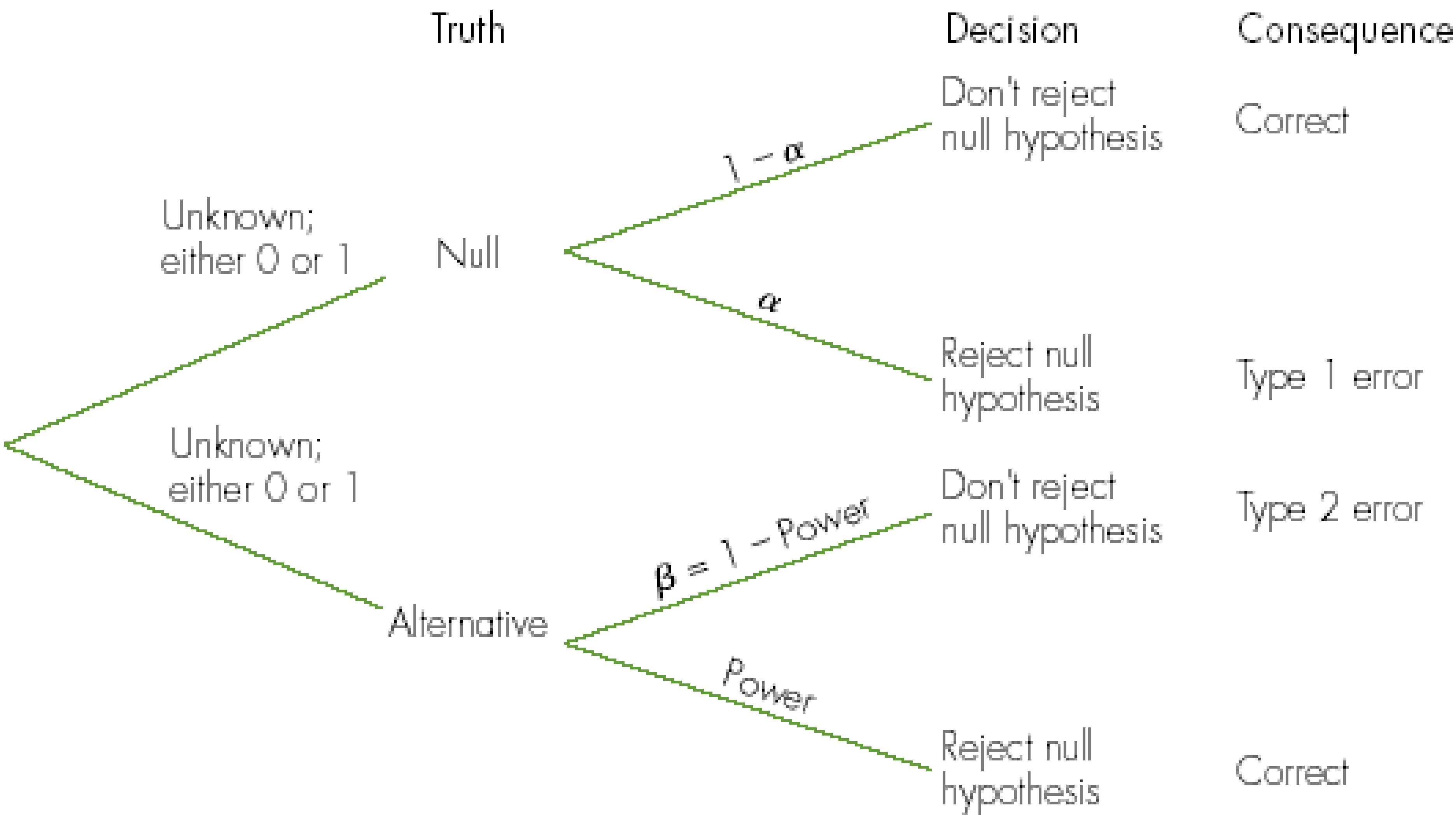
Power increases if:

- Sample size is increased (because having more evidence makes it easier to show that the alternative hypothesis is true, if it really is)
- The level of significance α is increased (because it's easier to reject H_0 when the rejection region is larger)
- The actual difference between the sample estimate and the null value increases (because it's easier to detect a true difference if it's large) *We have no control over this one!*

Trade-off must be taken into account when choosing α. If α is *small* it's *harder* to reject H_0. If α is *large* it's *easier* to reject H_0:

- If Type 1 error is *more* serious, use *smaller* α.
- If Type 2 error is *more* serious, use *larger* α.

Ways to picture the errors (see page 463):



Truth, decisions, consequences, *conditional* (row) probabilities:

Decision:

Truth:	**Don't reject H_0**	**Reject H_0**	**Error can occur**:
H_0	Correct $1 - \alpha$	Type 1 error α	Type 1 error can *only* occur if H_0 true
H_a	Type 2 error $\beta = 1 -$ power	Correct power	Type 2 error can *only* occur if H_a is true.
Error can occur:	Type 2, when H_0 not rejected	Type 1, when H_0 rejected	

SECTION 12.3: Test for difference in 2 proportions

Reminder from when we started Chapter 9

Five situations we will cover for the rest of this quarter:

Parameter name and description	**Population parameter**	**Sample statistic**
For Categorical Variables:		
One population proportion (or probability)	p	$\hat{p}$
Difference in two population proportions	$p_1 - p_2$	$\hat{p}_1 - \hat{p}_2$
For Quantitative Variables:		
One population mean	μ	$\bar{x}$
Population mean of paired differences (dependent samples, paired)	μ_d	$\bar{d}$
Difference in two population means (independent samples)	$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$

For each situation will we:

√ Learn about the *sampling distribution* for the sample statistic

√ Learn how to find a *confidence interval* for the true value of the parameter

- *Test hypotheses* about the true value of the parameter

Comparing two proportions from independent samples

Reminder on how we get independent samples (Lecture 19):

- **Random samples** taken separately from two populations and same response variable is recorded.

  <u>Example</u>: Compare proportions who think global warming is a problem, in two different years.

- **One random sample** taken and a variable recorded, but units are **categorized** to form two populations.

  <u>Example</u>: Compare 21 and over with under 21 for proportion who drink alcohol.

- Participants **randomly assigned** to one of two treatment conditions, and same response variable is recorded.

  <u>Example</u>: Compare aspirin and placebo groups for proportions who had heart attacks.

Hypothesis Test for Difference in Two Proportions

Example: (Source: http://www.pollingreport.com/enviro.htm)
Pew Research Center Poll taken in June 2006, just after the May release of *An Inconvenient Truth* (Al Gore's movie about global warming)

Asked 1500 people "In your view, is global warming a very serious problem, somewhat serious, not too serious, or not a problem?"

Results: 615/1500 = .41 or **41%** answered "Very serious"

Poll taken again with different 2001 people in November, 2011.

Results: 760/2001 = .38 or **38%** answered "Very serious."

Question: Did the *population* proportion that thinks it's very serious go down from 2006 to 2011, or is it chance fluctuation?

SIDE NOTE ABOUT THIS EXAMPLE:

They have not asked this question again since the 2011 poll, but in November 2012 a different poll asked this:

"If nothing is done to reduce global warming in the future, how serious of a problem do you think it will be for THE UNITED STATES: very serious, somewhat serious, not so serious, or not serious at all?"

This was a month after Hurricane Sandy.

Results: 49% said *very serious* and an additional 31% said *somewhat serious*. So clearly public opinion depends on when the poll is taken!

Notation and numbers for the Example:

Population parameter of interest is $p_1 - p_2$ where:

p_1 = proportion of all US adults in May 2006 who thought global warming was a very serious problem.

p_2 = proportion of all US adults in Nov 2011 who thought global warming was a very serious problem.

$\hat{p}_1$ = sample estimate from May 2006 = X_1/n_1 = 615/1500 = .41

$\hat{p}_2$ = sample estimate from Nov 2011 = X_2/n_2 = 760/2001 = .38

Sample statistic is $\hat{p}_1 - \hat{p}_2 = .41 - .38 = .03$

Five steps to hypothesis testing for difference in 2 proportions:
See ***Summary Box*** *on* ***page 479***

STEP 1: Determine the null and alternative hypotheses.

Null hypothesis is H_0: $p_1 - p_2 = 0$ (or $p_1 = p_2$); <u>null value = 0</u>

Alternative hypothesis is *one* of these, based on context:
H_a: $p_1 - p_2 \neq 0$ (or $p_1 \neq p_2$)
H_a: $p_1 - p_2 > 0$ (or $p_1 > p_2$)
H_a: $p_1 - p_2 < 0$ (or $p_1 < p_2$)

EXAMPLE:
Did the population proportion who think global warming is “very serious” drop from 2006 to 2011? This is the alternative hypothesis. (Note that it’s a <u>one-sided test</u>.)
H_0: $p_1 - p_2 = 0$ (no actual change in population proportions)
H_a: $p_1 - p_2 > 0$ (or $p_1 > p_2$; 2006 proportion > 2011 proportion)

STEP 2:
Verify data conditions. If met, summarize data into test statistic.

For Difference in Two Proportions:

Data conditions: $n\hat{p}$ and $n(1-\hat{p})$ are both at least 10 for both samples.

Test statistic:

$$z = \frac{\text{sample statistic} - \text{null value}}{\text{(null) standard error}}$$

Sample statistic $= \hat{p}_1 - \hat{p}_2$
Null value $= 0$
Null standard error:

- Computed *assuming* null hypothesis is true.
- If null hypothesis *is* true, then $p_1 = p_2$
- We get an estimate for the common value of p using *both samples*, then use that in the standard error formula. Details on next page.

$$\hat{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{\text{combined successes}}{\text{combined sample sizes}}$$

Null standard error = estimate of $\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$ using combined estimate $\hat{p}$ in place of both p_1 and p_2.

So the test statistic is:

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}}} = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}}$$

This “null standard error” is used because it’s the best way to estimate the standard deviation when in fact $p_1 = p_2$, i.e. when the null hypothesis is true.

Step 2 for the Example:

Data conditions are met, since both sample sizes are large.

$$\hat{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{\text{combined successes}}{\text{combined sample sizes}} = \frac{615 + 760}{1500 + 2001} = \frac{1375}{3501} = .39$$

Null standard error = $\sqrt{(.39)(1-.39)(\frac{1}{1500} + \frac{1}{2001})} = .0167$

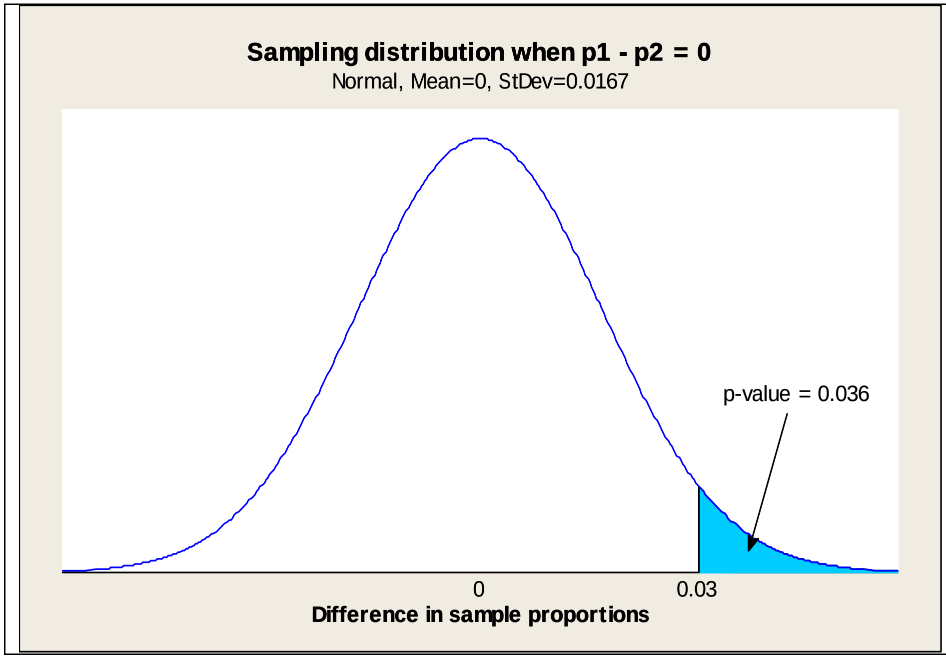
Test statistic:

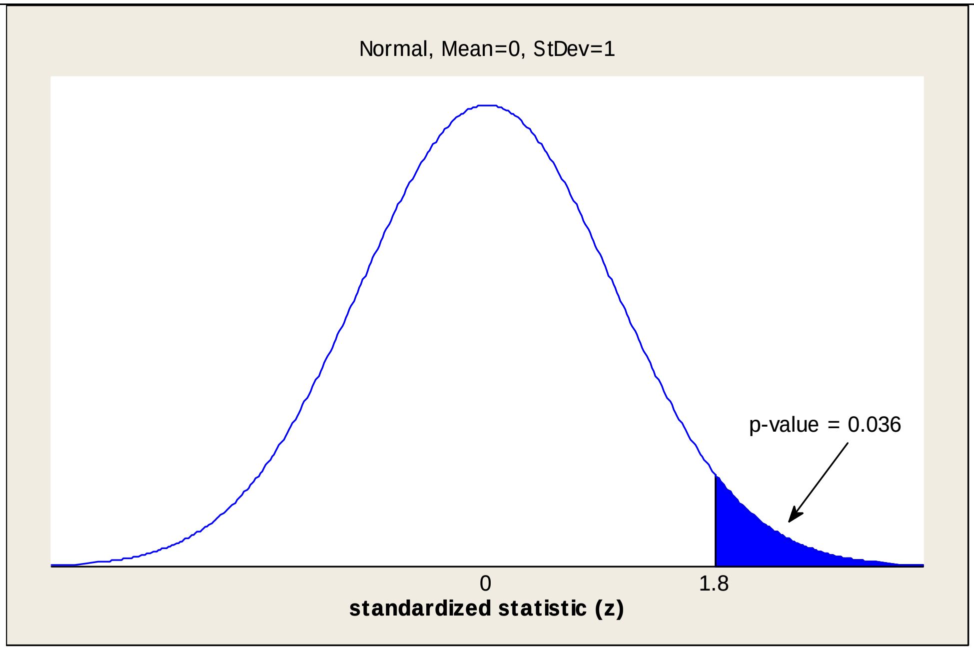
$$z = \frac{\text{sample statistic} - \text{null value}}{\text{(null) standard error}} = \frac{.03 - 0}{.0167} = 1.80$$

Pictures:

On left: Sampling distribution of $\hat{p}_1 - \hat{p}_2$ when population proportions are equal, showing where the observed value of $\hat{p}_1 - \hat{p}_2 = .03$ falls.

On right: Same picture, converted to z-scores, showing z = 1.80.





Area above $\hat{p}_1 - \hat{p}_2 = 0.03$ is .036

Area above z = 1.80, is 0.036.

STEP 3:
Assuming the null hypothesis is true, find the p-value.

General: *p*-value = the probability of a test statistic as extreme as the one observed or more so, in the direction of H_a, *if* the null hypothesis is true.

Difference in two proportions, same idea as one proportion. Depends on the alternative hypothesis. See pictures on p. **465**

Alternative hypothesis:	p-value is:
H_a: $p_1 - p_2 > 0$ (a one-sided hypothesis)	Area above the test statistic z
H_a: $p_1 - p_2 < 0$ (a one-sided hypothesis)	Area below the test statistic z
H_a: $p_1 - p_2 \neq 0$ (a two-sided hypothesis)	$2 \times$ the area above $\lvert z \rvert$ = area in tails beyond $-z$ and z

Example:
Alternative hypothesis is one-sided
H_a: $p_1 - p_2 > 0$
p-value = Area <u>above</u> the test statistic z = 1.80
From Table A.1, *p*-value = area above 1.80
= 1 – .9641 = .0359 ≈ .036.

STEP 4:
Decide whether or not the result is statistically significant based on the p-value.

Example: Use α of .05, as usual
p-value = .036 <.05, so:

- Reject the null hypothesis.
- Accept the alternative hypothesis
- The result is statistically significant

Step 5: Report the conclusion in the context of the situation.
Example:
Conclusion: From May 2006 to November 2011 there was a statistically significant decrease in the proportion of US adults who think global warming is "very serious."

Interpretation of the *p*-value (for this one-sided test):
It's a *conditional probability*. Conditional on the null hypothesis being true (equal population proportions), what is the probability that we would observe a *sample difference* as large as the one observed or larger just by chance?

Specific to this example: *If* there really were no change in the proportion of the population who think global warming is "very serious" what is the probability of observing a sample proportion in 2011 that is .03 (3%) or more lower than the sample proportion in 2006? Answer: The probability is .036. Therefore, we *reject the idea (the hypothesis)* that there was no change in the population proportion.