Today: Sections 13.1 to 13.3

ANNOUNCEMENTS:

• Discussion Friday is not for credit, review for final and help with this week’s homework. You may attend multiple discussions, but first priority goes to those registered for that discussion.

• Quiz for week 8 starts Fri, ends Monday at 10am.

• Final exam Monday – free response on new material only, multiple choice on all material from the quarter, mainly on concepts. Review sheets posted on website. Allowed 4 sheets of notes, both sides.

HOMEWORK (due Friday, March 15): Chapter 13
#14
#26 and 28 (connected, count together for 3 pts)
#54abcd (no computation needed for #54)

Hypothesis Testing for One Mean and Mean of Paired Differences (Same process, different notation)

Step 1: Determine the null and alternative hypotheses.

H₀: μ = μ₀
H₀: μₐ = 0
(Null value is called μ₀) (Null value is 0.)

Alternative hypothesis is one of these, based on context:

Hₐ: μ ≠ μ₀
Hₐ: μₐ ≠ 0
Hₐ: μ > μ₀
Hₐ: μₐ > 0
Hₐ: μ < μ₀
Hₐ: μₐ < 0

Step 2: Test statistic \( t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} \) (null) standard error

\[ t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} \]

STEP 3: Find the p-value. This is the only thing that’s new.

Finding p-value for test statistic \( t \) (instead of \( z \))

Same idea as other situations (see pictures on p. 503), but now we need to use the \( t \)-distribution with \( df = n - 1 \), instead of normal distribution.

For each situation will we:

✓ Learn about the sampling distribution for the sample statistic
✓ Learn how to find a confidence interval for the true value of the parameter
✓ Test hypotheses about the true value of the parameter
✓ For independent samples, will see how to do in R Commander only.

Finishing what we planned to cover when we started Chapter 9

Five situations we will cover for the rest of this quarter:

<table>
<thead>
<tr>
<th>Parameter name and description</th>
<th>Population parameter</th>
<th>Sample statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>For Categorical Variables:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>One population proportion (or probability)</td>
<td>( p )</td>
<td>( \hat{p} )</td>
</tr>
<tr>
<td>Difference in two population proportions</td>
<td>( p_1 - p_2 )</td>
<td>( \hat{p}_1 - \hat{p}_2 )</td>
</tr>
<tr>
<td>For Quantitative Variables:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>One population mean</td>
<td>( \mu )</td>
<td>( \bar{x} )</td>
</tr>
<tr>
<td>Population mean of paired differences (dependent samples, paired)</td>
<td>( \mu_d )</td>
<td>( \bar{d} )</td>
</tr>
<tr>
<td>Difference in two population means (independent samples)</td>
<td>( \mu_1 - \mu_2 )</td>
<td>( \bar{d} - \bar{d} )</td>
</tr>
</tbody>
</table>

For independent samples, will see how to do in R Commander only.
Use Table A.3 on page 671 (or can use Excel or R Commander) p-value for test of $H_0: \mu > \mu_0$ based on a $t$-statistic
Table will provide a $p$-value range, not an exact $p$-value.
Example: $t = 2.20$, $df = 9$. Across top, $2.00 < 2.20 < 2.33$
Use $df = 9$ row, columns for $2.00$ and $2.33$: $0.22 < p$-value $< 0.38$

Steps for finding the $p$-value when the test statistic is positive:
1. Compute the test statistic $t$.
2. Find the value(s) in Table A.3 that $t$ falls between (or above/below) and use those columns.
3. Go to the appropriate df row.
4. Read the $p$-value range from the table.
5. For $H_0$: parameter $> \mu_0$, you’re done.
6. For $H_0$: parameter $\neq \mu_0$, double the values.
7. For $H_0$: parameter $< \mu_0$, you need to subtract from 1.

Steps for finding the $p$-value when the test statistic is negative:
1. Do Steps 2 to 4 above using $|t|$
2. For $H_0$: parameter $< \mu_0$, you’re done.
3. For $H_0$: parameter $\neq \mu_0$, double the values.
4. For $H_0$: parameter $> \mu_0$, you need to subtract from 1.

The Rejection Region Approach (Easier!)
Rejection region is the set of test statistic values that will lead us to reject the null hypothesis. Refer to the bottom of Table A.2:

<table>
<thead>
<tr>
<th>$t$</th>
<th>1.20</th>
<th>1.50</th>
<th>1.65</th>
<th>1.80</th>
<th>2.00</th>
<th>2.33</th>
<th>2.50</th>
<th>2.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.111</td>
<td>.197</td>
<td>.237</td>
<td>.271</td>
<td>.308</td>
<td>.347</td>
<td>.383</td>
<td>.420</td>
</tr>
<tr>
<td>2</td>
<td>.144</td>
<td>.230</td>
<td>.270</td>
<td>.302</td>
<td>.334</td>
<td>.365</td>
<td>.400</td>
<td>.436</td>
</tr>
<tr>
<td>3</td>
<td>.185</td>
<td>.273</td>
<td>.309</td>
<td>.343</td>
<td>.375</td>
<td>.407</td>
<td>.439</td>
<td>.471</td>
</tr>
<tr>
<td>4</td>
<td>.232</td>
<td>.319</td>
<td>.354</td>
<td>.388</td>
<td>.420</td>
<td>.452</td>
<td>.484</td>
<td>.515</td>
</tr>
<tr>
<td>5</td>
<td>.286</td>
<td>.373</td>
<td>.408</td>
<td>.441</td>
<td>.472</td>
<td>.503</td>
<td>.535</td>
<td>.565</td>
</tr>
<tr>
<td>6</td>
<td>.346</td>
<td>.433</td>
<td>.467</td>
<td>.500</td>
<td>.531</td>
<td>.562</td>
<td>.593</td>
<td>.624</td>
</tr>
<tr>
<td>7</td>
<td>.414</td>
<td>.501</td>
<td>.534</td>
<td>.567</td>
<td>.598</td>
<td>.629</td>
<td>.660</td>
<td>.690</td>
</tr>
</tbody>
</table>

Using the portion of the table below, do these examples:
$H_0: \mu > \mu_0$, $t = 1.90$, $df = 7$. Result: $.043 < p < .057$
$H_0: \mu \neq \mu_0$, $t = 1.90$, $df = 7$. Result: $.086 < p < .114$
$H_0: \mu < \mu_0$, $t = 1.90$, $df = 7$. Result: $.943 < p < .957$
$H_0: \mu < \mu_0$, $t = -1.90$, $df = 7$. Result: $.043 < p < .057$
Rejection Region Approach, continued

Use Table A.2. (See full table on next page.)

- Find one-tailed or two-tailed $\alpha$ at the bottom of the table.
- Find appropriate df row.
- Read $t^*$ to form the rejection region (based on which $H_a$)

Examples based on segment of table below:

$H_0$: $\mu > \mu_0$, $\alpha = .05$, df = 100. Reject $H_0$ if test statistic $t \geq 1.66$.

$H_0$: $\mu < \mu_0$, $\alpha = .05$, df = 100. Rejection region is $t \leq -1.66$

$H_0$: $\mu \neq \mu_0$, $\alpha = .05$, df = 100. Rejection region is $t \leq -1.98$ and $t \geq 1.98$

<table>
<thead>
<tr>
<th>df</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
</tr>
<tr>
<td>90</td>
</tr>
<tr>
<td>100</td>
</tr>
<tr>
<td>1000</td>
</tr>
<tr>
<td>infinite</td>
</tr>
<tr>
<td>Two-tailed $\alpha$</td>
</tr>
<tr>
<td>.20</td>
</tr>
<tr>
<td>.10</td>
</tr>
<tr>
<td>.05</td>
</tr>
<tr>
<td>.02</td>
</tr>
<tr>
<td>One-tailed $\alpha$</td>
</tr>
<tr>
<td>.10</td>
</tr>
<tr>
<td>.05</td>
</tr>
<tr>
<td>.025</td>
</tr>
<tr>
<td>.01</td>
</tr>
</tbody>
</table>

### Summary of Notation and Tables for Student’s $t$-distribution

**Confidence intervals:**
- Notation for multiplier is $t^*$
- Use Table A.2 (page 670 or inside back cover) to find $t^*$

**Hypothesis tests:**
- Notation for the test statistic is $t$ (computed from data)
- $p$-value approach: After you compute $t$ from the data, use Table A.3 (page 671) to find the $p$-value range
- Rejection region approach
  - Compute $t$ from the data
  - Find the rejection region of form $t \geq t^*$, etc. (form depends on $H_0$); use Table A.2 to find $t^*$
  - See if the computed test statistic $t$ is in the rejection region

### Testing One Population Mean or Paired Differences

**Go through steps with an example**


For each person, $d_i = \text{difference in weight (after – before)}$ for people who quit smoking for 1 year. (Positive = weight gain)

$\mu_d = \text{population mean weight gain in 1 year for smokers who quit.}$

$H_0$: $\mu_d = 0$

$H_1$: $\mu_d \neq 0$

$n = 322$, Sample mean $= \bar{d} = 5.15$ pounds,

Sample standard deviation $= s_d = 11.45$ pounds

Standard error of $\bar{d} = \sqrt{\frac{s_d}{\sqrt{n}}} = \sqrt{\frac{11.45}{\sqrt{322}}} = .6381$
**STEP 2:**
Verify data conditions. If met, summarize data into test statistic.

**Data conditions:**
Bell-shaped data (no extreme outliers or skewness) or large sample.

**Test statistic (remember, use \( t \) for means):**

\[
t = \frac{\text{sample statistic} - \text{null value}}{\text{(null) standard error}}
\]

**One population mean:**

- **Sample statistic** = \( \bar{x} \)
- **Null value** = \( \mu_0 \)
- **Null standard error** = \( \frac{s}{\sqrt{n}} \)

**Mean of paired differences:**

- **Sample statistic** = \( \bar{d} \)
- **Null value** = 0
- **Null standard error** = \( \frac{s_d}{\sqrt{n}} \)

Note that the word “null” is unnecessary in std. error involving means.

**Step 2 for the Example:**
Data conditions are met, since sample size is large (\( n = 322 \)).

Population mean weight loss after quitting smoking = 0?

\[
t = \frac{\text{sample statistic} - \text{null value}}{\text{(null) standard error}} = \frac{5.15 - 0}{11.45 / \sqrt{322}} = \frac{5.15}{0.6381} = 8.07
\]

**p-value for our example of paired differences:**

Weight gain/loss when quitting smoking:

- \( H_0: \mu_d = 0 \)
- \( H_a: \mu_d \neq 0 \)
- \( t = 8.07 \)
- \( p\)-value = 2 \( \times \) P(sample mean \( \geq \) 5.15 pounds \| \) population mean = 0) = 2 \( \times \) area above \( |8.07| \) for \( t \)-distribution with \( df = 321 \).

Best we can do from Table A.3 is \( p\)-value < .004 (=2 \( \times \) .002)

- From Excel, \( p\)-value = \( 1.4 \times 10^{-14} \)
**STEP 4 – using p-values:** Decide whether or not the result is statistically significant based on the p-value.

**Example:**
Paired difference, mean weight gain/loss after quitting smoking:
p-value = $1.4 \times 10^{-14} < .05$, so:
- Reject the null hypothesis.
- Accept the alternative hypothesis
- The result is statistically significant

**STEP 5: Report the conclusion in the context of the situation.**

**Example:** The mean change in weight for one year after quitting smoking is significantly different from 0.

Note: A 95% confidence interval for the mean change in weight is: $5.15 \pm 1.97(.638)$ or 3.89 to 6.41 pounds.
Possible problem: No control group! People gain weight as they age.

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**Rejection Region Method (Substitute Steps 3 and 4):**

Use Table A.2 to find rejection region

$H_0: \mu_d = 0, \alpha = .05, \text{df} = 321$. Best we can do is use $\text{df} = 100$.

<table>
<thead>
<tr>
<th>df</th>
<th>1.29</th>
<th>1.66</th>
<th>1.99</th>
<th>2.37</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>1.29</td>
<td>1.66</td>
<td>1.99</td>
<td>2.37</td>
</tr>
<tr>
<td>90</td>
<td>1.29</td>
<td>1.66</td>
<td>1.98</td>
<td>2.36</td>
</tr>
<tr>
<td>100</td>
<td>1.29</td>
<td>1.66</td>
<td>1.98</td>
<td>2.36</td>
</tr>
<tr>
<td>1000</td>
<td>1.292</td>
<td>1.646</td>
<td>1.982</td>
<td>2.330</td>
</tr>
</tbody>
</table>

*Two-tailed a*  
| 0.20 | 0.10 | 0.05 | 0.02 |

*One-tailed a*  
| 0.10 | 0.05 | 0.025 | 0.01 |

Rejection region is $t \leq -1.98$ and $t \geq 1.98$

Test statistic $t = 8.07$ is clearly in the rejection region! So reject $H_0$.

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**Substitute Step 4: Rejection Region Approach**

If the test statistic is *not* in the rejection region:
- Do not reject the null hypothesis.
- There is not enough evidence to accept the alternative hypothesis
- The result is not statistically significant

If the test statistic *is* in the rejection region:
- Reject the null hypothesis.
- Accept the alternative hypothesis
- The result is statistically significant

For our example, the test statistic is definitely in the rejection region, so we reject the null hypothesis.
Hypothesis test for difference in means, independent samples

Called a “two-sample t-test” or “independent samples t-test.”

You already learned how to do this with R Commander.

Example from last time:
Two-sample t-test to compare study hours for those who drink and those who don’t. (Don’t confuse terms “two-sample” and “two-sided.”) Makes sense to do one-sided test.

Population 1:
Students who have or will take Stat 7, who don’t drink alcohol
μ₁ = mean hours of Stat 7 study for this population

Population 2:
Students who have or will take Stat 7, who drink alcohol
μ₂ = mean hours of Stat 7 study for this population

Research question: Is the mean hours of study per week for Stat 7 higher in the population who don’t drink alcohol than in the population who drink? (We know it’s higher in the sample.)

\[ H_0: \mu_1 = \mu_2 \quad \text{(i.e } H_0: \mu_1 - \mu_2 = 0) \]  \quad \text{Ha: } \mu_1 > \mu_2 \quad \text{(i.e. } H_a: \mu_1 - \mu_2 > 0) \]

• One column for Drink or Don’t drink, one for Study hours
• Statistics → Means → Independent samples t-test
• Choose the alternative (>) and confidence level

data: HoursStudy by Drink
\[ t = 1.9923, \text{ df = 70.556, } p-value = 0.02511 \]
alternative hypothesis: true difference in means is greater than 0
95 percent confidence interval:
0.2817279       Inf  [Note one-sided confidence interval for difference]
sample estimates:
mean in group Don’t drink       mean in group Drink
6.500000                  4.775735

\[ p-value = .02511 < .05, \text{ so reject } H_0 \text{ and conclude drinkers study fewer hours on average. C.I. gives lower bound as 0.28 hours.} \]

New Example (CS 13.1): Work through from start to finish

Research question: Can drinking an ice slushie increase endurance when exercising in hot weather?

Australia study published in Medicine and Science in Sports and Exercise, 2010

• 10 Male volunteers, average age 28
• Two treatments administered to all 10 men:
  ○ Drink fruit-flavored ice slushie
  ○ Drink fruit-flavored cold water
• Then run on treadmill in 93 degree room until exhausted
• Response variable = time until exhaustion
• Order randomized, administered a few weeks apart
• Did some practice runs to eliminate “learning effect”

Possible Errors for the Example

Type 1 error:
Slushies do not increase endurance, but this research study concludes that they do.

Consequence: People consume slushies to increase endurance, when cold water would have worked just as well.

Type 2 error:
Slushies do increase endurance, but this research study fails to discover that fact.

Consequence: People miss out on finding out about something that could help them increase endurance.
Parameter of interest: 
\( \mu_d \) = mean difference in exhaustion times if everyone in the population were to run under both conditions.

Hypotheses: 
H\(_0\): \( \mu_d = 0 \) (Slushie and water have same effect on endurance)  
H\(_a\): \( \mu_d > 0 \) (Slushie improves endurance)

Data and Test Statistic: 
\( \bar{d} = 9.5 \) minutes, \( s_d = 3.6 \) minutes, so s.e.\((\bar{d}) = \frac{3.6}{\sqrt{10}} = 1.14 \)

\[ t = \frac{9.5 - 0}{1.14} = 8.3, \text{ df} = 9, \text{ p-value} \approx 0. \]

Reject \( H_0 \), conclude ice slushie does increase endurance compared to drinking cold water.  
(Two-sided) 95% confidence interval: Multiplier \( t^* = 2.26 \)  
\( 9.5 \pm 2.26(1.14) \) or \( 6.9 \) to \( 12.1 \) minutes more, on average.