If this were the real exam, appropriate tables would be provided, but are not attached to this sample exam. Also, you would be given more space for some of the answers. This was condensed slightly to save paper.

1. In a study to investigate whether lack of sleep hurts short-term memory, 100 adults were each tested twice. Both times they studied a list of 30 words for two minutes, and were then asked to recall as many as possible, with a different but comparable list used each time. For one of the two tests they had been without sleep for 24 hours, while for the other they had a normal amount of sleep. The order was randomly assigned. The difference in number of words recalled (with sleep – without sleep) was recorded for each person. The average difference was 1.5 words, with a standard deviation of 6.1 words. Carry out the five steps of the hypothesis test for this situation, filling in the details in the space provided. Use $\alpha = 0.05$.

    Step 1: Determine the null and alternative hypotheses. (Be sure you define the parameter you use.)

    μ_d = mean difference in number of words people in the population would recall after normal sleep and lack of sleep, if everyone were to take the test
    $H_0: \mu_d = 0$
    $H_a: \mu_d > 0$

    Step 2: Verify necessary data conditions, and if met, summarize the data into an appropriate test statistic.

    Data conditions met because n = 100 and is thus large.
    Test statistic is $t = \dfrac{\bar{d} - 0}{\dfrac{s_d}{\sqrt{n}}} = \dfrac{1.5 - 0}{\dfrac{6.1}{\sqrt{100}}} = 2.46$

    Step 3: Assuming the null hypothesis is true, find the *p*-value.

    p-value = area above 2.46 for a t-distribution with df = 100 – 1 = 99.
    Using Table A.3, notice that 2.46 falls between 2.33 and 2.58, so the p-value is between 0.011 and 0.006 (using either df = 90 or df = 100). This is usually presented as 0.006 < p-value < 0.011.

    Step 4: Decide whether or not the result is statistically significant.

    The result is statistically significant because the p-value is less than 0.05.

    Step 5: Report the conclusion in the context of the situation.

    Reject the null hypothesis and accept the alternative hypothesis that the population mean difference in the number of words recalled before and after adequate sleep would be greater than 0. In other words, lack of sleep does seem to hurt short-term memory.

2. Refer to Question 1. Find a 95% confidence interval for the parameter you defined in Step 1 of that question. Write a sentence interpreting your interval.

    $\bar{d} \pm t^* \times \dfrac{s_d}{\sqrt{n}}$ *where t* = 1.98 (Table A.2, use df = 100). Result is* $1.5 \pm 1.98 \times \dfrac{6.1}{\sqrt{100}}$ *or 1.5±1.2 or 0.3 to 2.7.*

    We are 95% confident that people would recall an average of 0.3 to 2.7 fewer words from a list of 30 words when they had been without sleep for 24 hours, compared to having had adequate sleep.

3. Case Study 12.2 described an election that was challenged in court because there was suspicion of fraudulent use of absentee ballots. The judge had to decide whether to overturn the election and remove the winner from office, and would do so if he believed the hypothesis that fraud was involved. Although there were other possible reasons why the votes might differ from past elections, to simplify matters suppose fraud was the only possible reason. Therefore, the judge must decide between:

    H_0: No fraud was involved
    H_a: Fraud was involved

    The "summary statistic" used to test these hypotheses was the difference between Democrat and Republican votes in the absentee ballots. Expert witnesses presented evidence (based on results from prior elections and votes by machine ballot) that the p-value for testing these hypotheses is 0.06.

    a. Finish this sentence, correctly interpreting the p-value in this situation: *The p-value of .06 is the probability of observing an absentee ballot difference as extreme as the one observed or more so, under the assumption that* **no fraud was involved [i.e. the null hypothesis is true]**.

    b. Describe what a Type 1 error and its consequences would be in this situation.

    A Type 1 error would occur if no fraud was involved, but the judge concluded there was fraud and overturned the election. The consequence would be that the winner was unfairly removed from office.

    c. Describe what a Type 2 error and its consequences would be in this situation.

    A Type 2 error would occur if fraud was involved, but the judge was not convinced and did not overturn the election. The consequence would be that an official who was elected with fraudulent voting would be allowed to remain in office.

    d. The judge decided that fraud was involved and overturned the results of the election. Did the judge use a level of significance of 0.05? Explain how you know.

    No. We were told that the p-value was .06, yet the judge rejected the null hypothesis and accepted the alternative hypothesis. Therefore, he used a level of significance of .06 or more.

4. According to a recent news report "A new study confirms that kids who watch a lot of television tend to be heavier than kids who do not... kids who spent more than four hours in front of the tube were significantly heavier than children who watched fewer than two hours daily." Was this result based on paired data or on independent samples? Briefly explain how you know.

    It would have to be based on independent samples. There would be no natural way to pair kids who watch different amounts of TV.

5. For each of the following situations, give correct notation and define the parameter of interest, then write the correct notation and numerical value for the sample statistic.

    a. In a random sample of 7th grade students in California it was found that the average height of males and females differed by .92 inches, with males being taller.

       Parameter:

       $\mu_1 - \mu_2$ =Mean height of all 7th grade males in California – Mean height of all 7th grade females in California.

       Sample statistic:

       $\bar{x}_1 - \bar{x}_2 = .92$ inches

    b. An article from Reuters News (April 7, 1998) reported that a study of 6,100 school children randomly sampled from around the United States found that 14.9% had some hearing loss (low-frequency or high-frequency of at least 16 decibels).

       Parameter:

       p = the proportion of the population of school children in the United States who have some hearing loss.

       Sample statistic:

       $\hat{p} = .149$

6. An airline suspects that a majority of frequent fliers prefer aisle seats. They define p = proportion of the population of frequent fliers who prefer aisle seats, and plan to test the hypotheses H_0: p = .5 versus H_a: p > .5. The company statistician plans to conduct the test using α = 0.05 based on a random sample of 600 frequent fliers. Suppose in fact 55% of frequent fliers prefer aisle seats. Then the probability that the test will (correctly) result in rejecting the null hypothesis is .791. Based on this information, provide numerical values for each of the following.

    a. The null value = .5.

    b. The level of significance = 0.05.

    c. The power of the test = 0.791

    d. The probability of making a Type 1 error (think carefully about this one!) = 0 because a Type 1 error can only be made when the null hypothesis is true, and we have been told that the alternative hypothesis is true.

    e. The probability of making a Type 2 error = 1 – power = 1 – 0.791 = 0.209.