

SOLUTIONS TO STATISTICS 7 PRACTICE PROBLEMS FOR CHAPTER 7

Chapter 7: #12, 18, 26, 46, 52, 58, 59, 60, 62, 66, 93

- 7.12**
- a.** The relative frequency interpretation of probability applies here. The probability was most likely determined by observing the number of Americans injured by lightning during a number of years and dividing this by the average population in those years.
 - b.** The personal interpretation of probability applies here. The probability was determined from the neighbor's previous experience with tomato plants and her knowledge of the soil, sunlight and other conditions where her plants are grown.
 - c.** The relative frequency interpretation of probability applies here. The probability was determined by observing many, many properly cared for tomato plants, counting the number of plants that produced tomatoes, and dividing by the total number of plants observed.
 - d.** The relative frequency interpretation of probability applies here. The probability was determined by observing many U.S. couples and noting the proportion of couples in which the husband outlived the wife.
- 7.18** John's reasoning is not correct. In the long run, if he repeatedly plays the lottery, the proportion of times he would win is $1/1000$. This does not mean that he will definitely win once every 1000 times he tries.
- 7.26**
- a.** No. Mutually exclusive events with positive probability can never be independent. Because A and B are mutually exclusive events, $P(A \text{ and } B) = 0$. If A and B were independent, then $P(A \text{ and } B) = P(A)P(B) = (1/2)(1/3) = 1/6$, which is not the case in this problem. A different "proof" is that if we knew that A had occurred, then we would know that B could not have occurred because they are mutually exclusive. So $P(B|A) = 0$, which does not equal $P(B) = 1/3$.
 - b.** No. Probabilities for complementary events must add to 1 but $P(A) + P(B) = (1/2) + (1/3) = 5/6 \neq 1$.
- 7.46**
- a.** $P(A) = .55$; $P(A^c) = .45$; $P(B|A) = .80$; $P(B|A^c) = .10$.
 - b.** $P(A \text{ and } B) = P(A)P(B|A) = (.55)(.80) = .44$. This is the probability of being a Republican and voting for Candidate X.
 - c.** $P(A^c \text{ and } B) = P(A^c)P(B|A^c) = (.45)(.10) = .045$. This is the probability of being a non-Republican and voting for Candidate X.
 - d.** $P(B) = P(A \text{ and } B) + P(A^c \text{ and } B) = .485$.
 - e.** Candidate X received 48.5% of the votes.
- 7.52**
- a.** These probabilities were determined by observing the relative frequency. The travel planner observed a large number of those specific flights and recorded the proportion of those flights that arrived on time for the ship.
 - b.** Whether Harold's plane is on time is probably not independent of whether Maude's plane is on time. Bad weather conditions cause many flight delays. If there is bad weather and Harold's plane is delayed, there is probably a higher chance that Maude's plane will be delayed.

c. $P(\text{both arrive on time}) = (.8)(.9) = .72$. Use the multiplication rule for two independent events (Rule 3b).

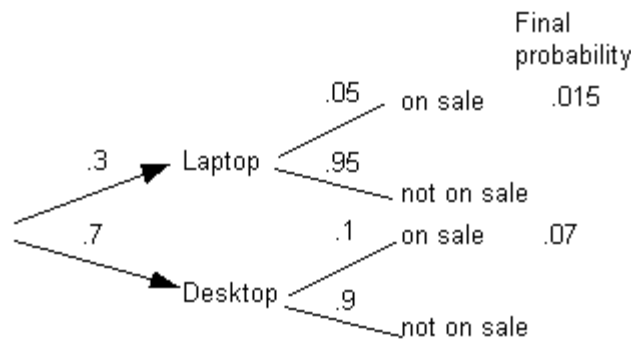
d. Each of the pair can be on time or late, so there are four mutually exclusive options, listed with their probabilities in the table below. The outcomes that result in one of them cruising alone are in bold. The outcomes are disjoint, so $P(\text{one cruises alone}) = .18 + .08 = .26$

	Maude is on time (.9)	Maude is late (.1)
Harold is on time (.8)	$(.8)(.9) = .72$	$(.8)(.1) = .08$
Harold is late (.2)	$(.2)(.9) = .18$	$(.2)(.1) = .01$

7.58 The tree diagram is below. The desired probability is the sum of the “final probabilities” for the two paths that end in “on sale.”

$$\begin{aligned}
 P(\text{on sale}) &= P(\text{laptop on sale}) + P(\text{desktop on sale}) \\
 &= P(\text{laptop}) P(\text{on sale} \mid \text{laptop}) + P(\text{desktop}) P(\text{on sale} \mid \text{desktop}) \\
 &= (.3)(.05) + (.7)(.1) = .015 + .070 = .085.
 \end{aligned}$$

Figure for Exercise 7.58



- 7.59 a. $P(A) = .80$; $P(A \text{ and } B) = .25$.
 b. $P(B|A) = P(A \text{ and } B)/P(A) = .25/.80 = .3125$.
 c. $P(B^c|A) = 1 - P(B|A) = 1 - .3125 = .6875$.

7.60 There are 49 possible pairs, of which 7 have the same number twice, so the probability is $7/49 = 1/7$. You could also reason that no matter what number the first student picks, the probability that the 2nd student picks that same number is $1/7$.

7.62 The table is as follows.

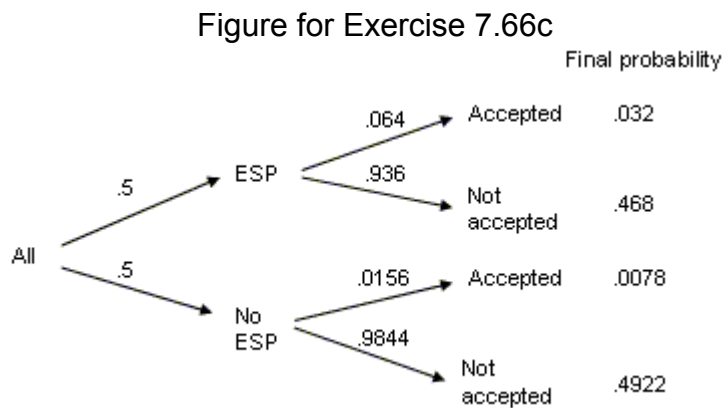
Grade	Calculus	No calculus	Total
Junior	15	35	50
Senior	25	25	50
Total	40	60	100

- 7.66**
- Probability = $(.25)(.25)(.25) = .0156$ that someone simply guessing gets three right and is accepted. Use the multiplication rule for independent events (Rule 3b extension).
 - Probability = $(.4)(.4)(.4) = .064$ that this person will be accepted for the later experiment. Use the multiplication rule for independent events (Rule 3b extension).
 - The desired conditional probability is $P(\text{ESP} \mid \text{accepted for experiment})$.

$$P(\text{ESP} \mid \text{accepted for experiment}) = \frac{P(\text{ESP and accepted})}{P(\text{accepted})} = \frac{.032}{.032 + .0078} = \frac{.032}{.0398} = .804.$$

This is an application of Rule 4 for conditional probability.

A tree diagram is useful for seeing the steps here. The first set of branches represents ESP or no ESP and the second set of branches represents being accepted or not within each ESP category.



Note that a participant who is accepted either really has ESP or does not. So (using Rule 2b) $P(\text{accepted}) = P(\text{ESP and accepted}) + P(\text{no ESP and accepted})$

Each element of the previous formula can be found using the multiplication rule (Rule 3a).

$$P(\text{ESP and accepted}) = P(\text{ESP}) \times P(\text{accepted} \mid \text{ESP}) = (.5)(.064) = .032$$

$$P(\text{no ESP and accepted}) = P(\text{no ESP}) \times P(\text{accepted} \mid \text{no ESP}) = (.5)(.0156) = .0078$$

This leads to $P(\text{accepted}) = .032 + .0078 = .0398$.

$$\text{So, } P(\text{ESP} \mid \text{accepted}) = \frac{P(\text{ESP and accepted})}{P(\text{accepted})} = \frac{.032}{.0398} = .804.$$

- 7.93**
- $P(\text{no failure}) = 1 - P(\text{failure}) = 1 - (1/10,000) = 9,999 / 10,000 = .9999$.
 - $P(\text{no failures among 4 plugs}) = .9999^4 = .9996$. This is an application of the multiplication rule for independent events. We are finding the probability that the first *and* second *and* third *and* fourth spark plugs do not fail.
 - $P(\text{at least one fails}) = 1 - P(\text{no failures}) = 1 - .9996 = .0004$.