Probability: Psychological Influences and Flawed Intuitive Judgments

(Section 7.7 and more)

Thought Question 1:

What is wrong with the following statement?

"The probability that you will die from a bee sting is about 15 times higher than the probability that you will die from a shark attack."

Specific People versus Random Individuals

Are you allergic to bees?
Do you swim where there are sharks?

On average, about 60 people in the US die of bee stings per year.
On average, about 4 people die from shark attacks.
But what about ***you personally***?

Two correct ways to express the aggregate statistics:

- *In the long run*, about 15 times more people die from bee stings than from shark attacks.
- *A randomly selected death* is about 15 times more likely to have occurred from a bee sting than from a shark attack.

Thought Question 2:

Do you think it is ***likely*** that anyone will ever ***win the multi-million dollar state lottery twice*** in a lifetime?

Coincidences

Are Coincidences Improbable?

A coincidence is a surprising concurrence of events, perceived as meaningfully related, with no apparent causal connection. (*Source:* Diaconis and Mosteller, 1989, p. 853)

Example 7.32: Winning the Lottery Twice

- NYT story of February 14, 1986, about Evelyn Marie Adams, who won the NJ lottery twice in short time period.
- NYT claimed that the odds of one person winning the top prize twice were about 1 in 17 trillion.

Source: Moore (1991, p. 278)

Someone, Somewhere, Someday

What is ***not improbable*** *is that someone, somewhere, someday will experience those events or something similar.*

We often ask the wrong question …

- The ***1 in 17 trillion*** is the probability that a ***<u>specific</u>*** individual who plays the NJ state lottery exactly twice will win both times (Diaconis and Mosteller, 1989,p. 859).
- ***Millions of people play lottery*** every day, so not surprising that someone, somewhere, someday would win twice.
- Stephen Samuels and George McCabe calculated … ***at least a 1 in 30 chance*** of a double winner in a 4-month period and better than even odds that there would be a double winner in a 7-year period somewhere in the U.S.

Consider these coincidences – can they be explained? What is the probability of…

- Flying from London to Sweden your professor ran into someone she knew in the gate area. Not only that, but it turned out that they had been assigned seats next to each other.
- Your professor was visiting New York City with a friend, and just happened to mention someone who had gone to college with her, who she hadn't seen for years. Five minutes later, she ran into that person. The person also said she was just thinking about her too!
- Someone dreams of a plane crash, and the next day one happens.

Most Coincidences Only Seem Improbable

- Coincidences seem improbable only if we ask the probability of that ***specific event occurring at that time to us***.
- If we ask the probability of it occurring some time, to someone, the probability can become quite large.
- Multitude of experiences we have each day => not surprising that ***some*** may appear ***improbable***.

Thought Question 3 (revisited):

You test positive for rare disease for which your original chances of having disease are 1 in 1000.

The test has a 10% false positive rate and a 10% false negative rate => ***whether you have disease or not, test is 90% likely to give a correct answer***.

Given you tested positive, what do you think is the ***probability that you actually have disease***? Higher or lower than 50%?

In other words, we know P(pos. test | disease) and want P(disease | pos. test)

Psychologists call this "Confusion of the Inverse" - Confusing P(A|B) with P(B|A)

The Probability of False Positives

If *base rate* for disease is very low and test for disease is less than perfect, there will be a relatively high probability that a positive test result is a ***false positive***.

To determine probability of a positive test result being accurate, you need:

1. **Base rate** – the probability that someone like you is likely to have the disease, without any knowledge of your test results.
2. **Sensitivity** of the test – the proportion of people who correctly test positive when they actually have the disease
3. **Specificity** of the test – the proportion of people who correctly test negative when they don't have the disease

Use tree diagram, hypothetical 100,000 or Bayes' Rule (p. 252).

Thought Question Calculation

Base rate = 1/1000, Sensitivity = .90, Specificity = .90

	Test pos.	Test neg.	Total
Actually sick	90	10	100
Actually healthy	9990	89,910	99,900
Total	10,080	89,920	100,000

So, P(Actually sick | Test positive) = 90/10080 = 0.9% or about 9/1000, even though P(Test pos. | Sick) = .90 or 9/10

Another Example: How dangerous are cell phones when driving?

- 2001 report found the probability that a driver who had an accident had been talking on a cell phone was only .015 (1.5%), whereas the probability that they were distracted by another occupant in the car was .109 (10.9%). Led cell phone proponents to say they weren't a problem.
- But that was P(Cell phone | Accident)
- What we really want is P(Accident | Cell phone), much harder to find, because we don't know P(Cell phone) = proportion on cell phone while driving!

Thought Question 4:

If you were to flip a fair coin six times, which sequence do you think would be most likely:

HHHHHH or HHTHTH or HHHTTT?

The Gambler's Fallacy

People think the long-run frequency of an event should apply even in the short run.

Tversky and Kahneman (1982) call it **belief in the law of small numbers,** *"according to which [people believe that] even small samples are highly representative of the populations from which they are drawn."* … *"in considering tosses of a coin for heads and tails … people regard the sequence HTHTTH to be more likely than the sequence HHHTTT, which does not appear to be random, and also more likely than HHHHTH, which does not represent the fairness of the coin"*

The Gambler's Fallacy

Independent Chance Events Have No Memory – they are not "self-correcting!"

Example:
People tend to believe that a string of good luck will follow a string of bad luck in a casino. However, making ten bad gambles in a row doesn't change the probability that the next gamble will also be bad.

The Gambler's Fallacy

When It May Not Apply

The Gambler's fallacy applies to independent events. It may not apply to situations where knowledge of one outcome affects probabilities of the next.

Example:
In card games using a single deck, knowledge of what cards have already been played provides information about what cards are likely to be played next.

Thought Question 5:

Which one would you choose in each set? (Choose either A or B and either C or D.) Explain your answer.

A. A gift of $240, guaranteed
B. A 25% chance to win $1000 and a 75% chance of getting nothing.

C. A sure loss of $740
D. A 75% chance to lose $1000 and a 25% chance to lose nothing

Using Expected Values To Make Wise Decisions

If you were faced with the following alternatives, **which would you choose?** Note that you can choose **either A or B and either C or D**.
A. A gift of $240, guaranteed
B. A 25% chance to win $1000 and a 75% chance of getting nothing
C. A sure loss of $740
D. A 75% chance to lose $1000 and a 25% chance to lose nothing

- **A versus B:** majority chose *sure gain* A. Expected value under choice B is $250, higher than sure gain of $240 in A, yet people prefer A.
- **C versus D:** majority chose *gamble* rather than *sure loss*. Expected value under D is $750, a larger expected loss than $740 in C.
- People **value sure gain**, but willing to **take risk to prevent loss**.
- But, depends on $$values!

Source: Plous (1993, p. 132)

Using Expected Values: Depends on how much is at stake!

If you were faced with the following alternatives, **which would you choose?** Note that you can choose **either A or B and either C or D**.
Alternative A: A 1 in 1000 chance of winning $5000
Alternative B: A sure gain of $5
Alternative C: A 1 in 1000 chance of losing $5000
Alternative D: A sure loss of $5

- **A versus B:** Now 75% chose A (*gamble*). Similar to decision to buy a lottery tickets, where sure gain is keeping $5 rather than buy 5 tickets.
- **C versus D:** 80% chose *sure loss* D rather than *gamble*. Similar to buying insurance. Dollar amounts are important: sure loss of $5 easy to absorb, while risk of losing $5000 may risk bankruptcy.

Source: Plous (1993, p. 132)

Psychological Issues on Reducing Risk

Certainty Effect: people more willing to pay to reduce risk from fixed amount down to 0 than to reduce risk by same amount when not reduced to 0.

Example: Probabilistic Insurance
- Students asked if want to buy "probabilistic insurance" … ***costs half as much as regular insurance but only covers losses with 50% probability***.
- Majority (80%) not interested.
- Expected value for return is same as regular policy.
- Lack of assurance of payoff makes it unattractive.

Source: Kahneman and Tversky

Pseudocertainty Effect: people more willing to accept a complete reduction of risk on certain problems and no reduction on others than to accept a reduced risk on a variety (all) problems.

Example: Vaccination Questionnaires
- ***Form 1: probabilistic protection*** = vaccine available for disease (e.g. flu) that afflicts 20% of population but would protect with 50% probability. **40% would take vaccine.**
- ***Form 2: pseudocertainty*** = two strains, each afflicting 10% of population; vaccine completely effective against one but no protection from other. **57% would take vaccine**.
- In both, vaccine reduces risk from 20% to 10% but complete elimination of risk perceived more favorably.

Source: Slovic, Fischhoff, and Lichtenstein, 1982, p. 480.

Thought Question 6:

Which do you think caused more deaths in the United States in 2005*, homicide or diabetes? What do you think the ratio was?

*Latest year for which data are available

Assessing Personal Probability in repeatable and non-repeatable situations

- **Personal probabilities**: values assigned by individuals based on how likely they think events are to occur
- Some situations are not repeatable, or you may not have the data.
- Still should ***follow the rules*** of probability.
- But our intuition doesn't seem to know or understand those rules!

Psychologists have defined "heuristics" about probability

The Availability Heuristic (Tversky and Kahneman, 1982): "there are situations in which people assess the probability of an event by the ease with which instances or occurrences can be brought to mind. This judgmental heuristic is called ***availability***."

Which do you think caused more deaths in the United States in 2005, homicide or diabetes?

Most answer *homicide*. The actual 2005 numbers were 18,124 homicides and 75,119 deaths from diabetes. (Ratio > 4.)

Distorted view that homicide is more probable results from the fact that ***homicide receives more attention in the media***.

Detailed Imagination – An example of using Availability

Lawyers use this trick with juries…
Risk perceptions distorted by having people vividly imagine an event – how the crime *could* have occurred.

Another Example:
Salespeople convince you that $500 is a reasonable price to pay for an extended warranty on your new car by having you imagine that if your air conditioner fails it will cost you more than the price of the policy to get it fixed. They don't mention that it is extremely unlikely that your air conditioner will fail during the period of the extended warranty.

Another heuristic: Anchoring

Risk perception distorted by providing a reference point, or **anchor**, from which people adjust up or down. Most tend to stay close to the anchor provided.

Anchoring Example: **(Exercise 13.75)** Two groups of students asked to estimate the population of Canada:

- **High-anchor** version: "The population of the U.S. is about 270 million. To the nearest million, what do you think is the population of Canada?" ***mean* = 88.4**
- **Low-anchor** version: "The population of Australia is about 18 million. To the nearest million, what do you think is the population of Canada?" ***mean* = 22.5**

(It was actually slightly over 30 million at that time.)

Thought Question 7:

Plous (1993) presented readers with the following test: Place a check mark beside the alternative that **seems most likely to occur within the next 10 years**:

- An all-out nuclear war between the United States and Russia
- An all-out nuclear war between the United States and Russia in which neither country intends to use nuclear weapons, but both sides are drawn into the conflict by the actions of a country such as Iraq, Libya, Israel, or Pakistan.

Using your intuition, pick the more likely event at that time.

The Representativeness Heuristic and the Conjunction Fallacy

Representativeness heuristic: People assign higher probabilities than warranted to scenarios that are *representative* of how we *imagine* things would happen.

This leads to the **conjunction fallacy** … when detailed scenarios involving the conjunction of events are given, people assign higher probability assessments to the combined event than to statements of one of the simple events alone.

Remember that *P(A <u>and</u> B) cannot <u>exceed</u> P(A)*.

An Active Bank Teller

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in antinuclear demonstrations.

Respondents asked which of two statements is ***more probable***:

1. Linda is a bank teller.
2. Linda is a bank teller who is active in the feminist movement.

Results: "in a large sample of statistically naïve undergraduates, 86% judged the second statement to be more probable".

Problem: If Linda falls into the second group, she must also fall into the first group (bank tellers). Therefore, the first statement *must* have a higher probability of being true.

Source: Kahneman and Tversky (1982, p. 496)

Thought Question 8:

A fraternity consists of 30% freshmen and sophomores and 70% juniors and seniors. ***Bill is a member of the fraternity, he studies hard, he is well-liked by his fellow fraternity members, and he will probably be quite successful when he graduates***.

Is there any way to tell if Bill is **more likely** to be a lower classman (freshman or sophomore) or an upper classman (junior or senior)?

Forgotten Base Rates

The representativeness heuristic can lead people to ignore information about the likelihood of various outcomes.

Example:

People were told a population has 30 engineers and 70 lawyers. Asked: What is the likelihood that a randomly selected individual would be an engineer? Average close to 30%. Subjects given description below and again asked likelihood.

> *Dick is a 30-year-old man. He is married with no children. A man of high ability and high motivation, he promises to be quite successful in his field. He is well liked by his colleagues.*

Subjects ignored base rate of 30%, median response was 50%. Because he was randomly selected, probability of engineer = .3

Source: Kahneman and Tversky (1973, p. 243)

Optimism, Reluctance to Change, and Overconfidence

Optimism

Slovic and colleagues (1982, pp. 469–470) note that "the great majority of individuals believe themselves to be better than average drivers, more likely to live past 80, less likely than average to be harmed by the products they use, and so on."

Example: Optimistic College Students

On the average, students rated themselves as 15 percent more likely than others to experience positive events and 20 percent less likely to experience negative events.

Sources: Weinstein (1980) and Plous (1993, p. 135)

Reluctance to Change

The reluctance to change one's personal-probability assessment or belief based on new evidence.

Plous (1993) notes, "***Conservatism*** is the tendency to change previous probability estimates more slowly than warranted by new data" (p. 138).

Overconfidence

The tendency for people to place too much confidence in their own assessments. When people venture a guess about something for which they are uncertain, they tend to overestimate the probability that they are correct.

Example: How Accurate Are You?

Study Details:
Asked people hundreds of questions on general knowledge. e.g. Does *Time* or *Playboy* have a larger circulation? Also asked to rate odds they were correct, from 1:1 (50% probability) to 1,000,000:1 (virtually certain).

Results: the more confident the respondents were, the more the true proportion of correct answers *deviated* from the odds given by the respondents.

Solution: Plous (1993, p. 228) notes, "The most effective way to improve calibration seems to be very simple: *Stop to consider reasons why your judgment might be wrong*".

Source: Fischhoff, Slovic, and Lichtenstein (1977)

Calibrating Personal Probabilities of Experts

Professionals who help others make decisions (doctors, meteorologists) often use personal probabilities themselves.

Using Relative Frequency to Check Personal Probabilities

For a ***perfectly calibrated*** weather forecaster, of the many times they gave a 30% chance of rain, it would rain 30% of the time. Of the many times they gave a 90% chance of rain, it would rain 90% of the time, etc.

We can assess whether probabilities are **well-calibrated** only if we have enough repetitions of the event to apply the relative-frequency definition.

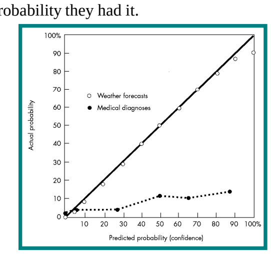
Calibrating Weather Forecasters and Physicians (from Seeing Through Statistics)

Open circles: actual relative frequencies of rain vs. forecast probabilities.
Dark circles relative frequency patient actually had pneumonia vs. physician's personal probability they had it.

Weather forecasters were quite accurate, well calibrated.

Physicians tend to overestimate the probability of disease, especially when the baseline risk is low.

When your physician quotes a probability, ask "is it a personal probability or based on data?"



Source: Plous, 1993, p. 223

Tips for Improving Personal Probabilities and Judgments

1. Think of the ***big picture***, including risks and rewards that are not presented to you. For example, when comparing insurance policies, be sure to compare coverage as well as cost.
2. When considering how a decision changes your risk, try to ***find out what the baseline risk is*** to begin with. Try to determine risks on an equal scale, such as the drop in *number* of deaths per 100,000 people rather than the *percent* drop in death rate.

Tips for Improving Personal Probabilities and Judgments

3. Don't be fooled by ***highly detailed scenarios***. Remember that excess detail actually decreases the probability that something is true, yet the representativeness heuristic leads people to increase their personal probability that it is true.
4. Remember to list reasons why your judgment might be wrong, to provide a more realistic confidence assessment.

Tips for Improving Personal Probabilities and Judgments

5. Do not fall into the trap of thinking that bad things only happen to other people. ***Try to be realistic*** in assessing your own individual risks, and make decisions accordingly.
6. Be aware that the techniques discussed here are often used in marketing. For example, ***watch out for the anchoring effect*** when someone tries to anchor your personal assessment to an unrealistically high or low value.
7. If possible, ***break events into pieces*** and try to assess probabilities using the information in Chapter 7 and in publicly available information.