Announcements:
• Slight change in schedule: We will cover 8.1 to 8.3 on Monday, then 7.7 on Wed, because you need 8.1 to 8.3 for Discussion.
• If you haven’t taken the survey yet, I encourage you to do it this weekend!
• Chapter 7 practice problems are now on website.
Homework (due Wed, Oct 27th):
Chapter 7:
#35, 51, 53 (answer in back, show work), 55

Chapter 7

Probability continued:
Finish slides from Wed first

Conditional Probabilities
The conditional probability of the event B, given that the event A will occur or has occurred, is the long-run relative frequency with which event B occurs when circumstances are such that A also occurs; written as $P(B|A)$.

$P(B) =$ unconditional probability event B occurs.

$P(B|A) =$ “probability of B given A”
= conditional probability event B occurs given that we know A has occurred or will occur.

EXAMPLE OF CONDITIONAL PROBABILITY

<table>
<thead>
<tr>
<th>Sleep style</th>
<th>No Myopia</th>
<th>Myopia</th>
<th>High Myopia</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Darkness</td>
<td>125 (66%)</td>
<td>15 (8%)</td>
<td>2 (1%)</td>
<td>142</td>
</tr>
<tr>
<td>Nightlight</td>
<td>110 (66%)</td>
<td>20 (11%)</td>
<td>2 (1%)</td>
<td>132</td>
</tr>
<tr>
<td>Full Light</td>
<td>36 (46%)</td>
<td>36 (46%)</td>
<td>5 (7%)</td>
<td>77</td>
</tr>
<tr>
<td>Total</td>
<td>371</td>
<td>71</td>
<td>9 (7%)</td>
<td>451</td>
</tr>
</tbody>
</table>

Random circumstance: Observe one randomly selected child
A = child slept in darkness as infant [Use “total” column.]
$P(A) = 172/479 = .36$
B = child did not develop myopia [Use “total” row]
$P(B) = 342/479 = .71$
$P(B|A) = P(\text{no myopia | slept in dark}) [Use “darkness” row]$
= 155/172 = .90 \neq P(B)$

NOTES ABOUT CONDITIONAL PROBABILITY
1. $P(B|A)$ generally does not equal $P(B)$.
2. $P(B|A) = P(B)$ only when A and B are independent events
3. In Chapter 6, we were actually testing if two types of events were independent.
4. Conditional probabilities are similar to row and column proportions (percents) in contingency tables. (Myopia example on previous page.)

Review:
RELATIONSHIPS BETWEEN EVENTS
• Defined for events in the same random circumstance only:
  – Complement of an event A is all simple events not part of A.
  – Mutually exclusive = disjoint events have no overlapping simple events
  – Complements are disjoint events (no overlapping simple events)
  – Disjoint events aren’t complements unless they cover all possibilities.
Review: Independent and Dependent Events

- Two events are **independent** of each other if knowing that one will occur (or has occurred) does not change the probability that the other occurs.
- Two events are **dependent** if knowing that one will occur (or has occurred) changes the probability that the other occurs.

The definitions can apply either...

to events within the same random circumstance or to events from two separate random circumstances.

**HOW TO DETERMINE IF TWO EVENTS ARE INDEPENDENT**

1. Physical assumption
   *Example*: Lottery draws on different days don’t affect each other.

2. See if \( P(B\mid A) = P(B) \). If so, \( A \) and \( B \) are independent events, otherwise they are not.
   *Example*: Suppose data showed that smokers and non-smokers are equally likely to get the flu.
   \( P(\text{flu} \mid \text{smoker}) = P(\text{flu}) \) so they are independent.

3. Events \( A \) and \( B \) are independent if and only if \( P(A \text{ and } B) = P(A)P(B) \). Sometimes have info so you can check to see if this is true.

7.4 Basic Rules for Finding Probabilities

**Probability an Event Does Not Occur**

Rule 1 (for “not the event”): \( P(A^c) = 1 – P(A) \)

*Example*: In the US, blood type percents* are:

- O: 46%,
- A: 39%,
- B: 11%,
- AB: 4%

Suppose we randomly select someone.

\[ P(\text{Blood type O}) = 0.46 \]

so \( P(\text{Blood type A, B, or AB}) = 1 – 0.46 = 0.54. \)

*Source: www.bloodbook.com

**Rule 2 (addition rule for “either/or/both”):**

*Rule 2a (general):*

\[ P(A \text{ or } B) = P(A) + P(B) – P(A \text{ and } B) \]

*Rule 2b (for mutually exclusive events):*

\[ P(A \text{ or } B) = P(A) + P(B) \]

Illustrate with Venn diagram on board.

**Example 7.14 Roommate Compatibility**

Brett is off to college and will be assigned a roommate from 1000 other males. Will he get one who snores? Parties? Both?

<table>
<thead>
<tr>
<th>Snores?</th>
<th>Likes to Party</th>
<th>Yes</th>
<th>No</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td></td>
<td>150</td>
<td>100</td>
<td>250</td>
</tr>
<tr>
<td>No</td>
<td></td>
<td>200</td>
<td>550</td>
<td>750</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>350</td>
<td>650</td>
<td>1000</td>
</tr>
</tbody>
</table>

\[ P(A) = \frac{250}{1000} = 0.25 \]

\[ P(B) = \frac{350}{1000} = 0.35 \]

\[ P(A \text{ and } B) = \frac{150}{1000} = 0.15 \]

Probability Brett will be assigned a roommate who either likes to party or snores, or both is:

\[ P(A \text{ or } B) = P(A) + P(B) – P(A \text{ and } B) = 0.25 + 0.35 – 0.15 = 0.45 \]

So the probability his roommate is acceptable is \( 1 – 0.45 = 0.55 \).
Rule 3 (multiplication rule for “and”):

- Rule 3a (general): \( P(A \text{ and } B) = P(A)P(B|A) \)
- Rule 3b (for independent events):
  \[ P(A \text{ and } B) = P(A)P(B) \]

Extension of Rule 3b (> 2 independent events):

For several independent events, \( P(A_1 \text{ and } A_2 \text{ and } \ldots \text{ and } A_n) = P(A_1)P(A_2)\ldots P(A_n) \)

Example with independent events (Rule 3b)

The probability of a birth being a boy is .512. Suppose a woman has 2 kids.

- \( A \): first child is a boy
- \( B \): second child is a boy

We assume these are independent events.

\[ P(A \text{ and } B) = P(A)P(B) = (.512)(.512) = .2621 \]

From Rule 2b, probability of same sex = .5002
From Rule 1, probability of different sex = .4998

Example with dependent events (Rule 3a)

Randomly select a student who takes Stat 8 with me.

- \( A \): student comes to class regularly; \( P(A) = 0.7 \)
- \( A^C \): student doesn’t come regularly; \( P(A^C) = 0.3 \)
- \( B \): student gets at least a B in the course

\[ P(B|A) = 0.8 \quad P(B|A^C) = 0.4 \]

What is the probability that the student comes to class regularly and gets at least a B?

\[ P(A \text{ and } B) = P(A)P(B|A) = (.7)(.8) = .56 \]

Determining a Conditional Probability

Rule 4 (conditional probability):

\[ P(B|A) = \frac{P(A \text{ and } B)}{P(A)} \]

\[ P(A|B) = \frac{P(A \text{ and } B)}{P(B)} \]

Blood Types Again

A blood bank needs Type O blood. What is the probability that the next 3 donors (not related) all have Type O blood? We can assume independent.

- Event \( A \): 1st donor has Type O blood, \( P(A) = .46 \)
- Event \( B \): 2nd donor has Type O blood, \( P(B) = .46 \)
- Event \( C \): 3rd donor has Type O blood, \( P(C) = .46 \)

\[ P(\text{Next 3 donors all have Type O blood}) = P(A \text{ and } B \text{ and } C) = (.46)(.46)(.46) = .097336 \]
Mutually exclusive versus Independent

Students sometimes confuse the definitions of independent and mutually exclusive events.

• When two events are mutually exclusive and one happens, it turns the probability of the other one to 0.
• When two events are independent and one happens, it leaves the probability of the other one alone.

In Summary (see page 249) …

| When Events Are | P(A or B) icc | P(A and B) icc | P(A|B) icc |
|-----------------|---------------|---------------|-----------|
| Mutually Excl.   | P(A) + P(B)   | 0             | 0         |
| Independent     | P(A) + P(B) – P(A|B) | P(A|B) | P(A) |
| Boy             | P(A) + P(B) – P(A and B) | P(A|B) | P(A and B) |

Steps for Finding Probabilities

Step 1: List each separate random circumstance involved in the problem.
Step 2: List the possible outcomes for each random circumstance.
Step 3: Assign whatever probabilities you can with the knowledge you have.
Step 4: Specify the event for which you want to determine the probability.
Step 5: Determine which of the probabilities from Step 3 and which probability rules can be combined to find the probability of interest.

7.5 Strategies for Finding Complicated Probabilities

Example 7.20 Winning the Daily 3 Lottery
Event A = winning number is 956. What is P(A)?
Method 1: With physical assumption that all 1000 possibilities are equally likely, P(A) = 1/1000.
Method 2: Define three events, B1 = 1st digit is 9, B2 = 2nd digit is 5, B3 = 3rd digit is 6
Event A occurs if and only if all 3 of these events occur. Note: P(B1) = P(B2) = P(B3) = 1/10. Since these events are all independent, we have P(A) = (1/10)^3 = 1/1000.

* Can be more than one way to find a probability.

Some Hints for Finding Probabilities

• P(A and B): Sometimes you can define the event in physical terms and know the probability or find it from a two-way table.
Example: I could classify the class into male, female and also year in school. Then, for example, probability that a randomly selected student in the class is Male and sophomore is the proportion of the class in that cell of the table. Don’t need separate P(A) and P(B).
• Check if probability of the complement is easier to find, then subtract it from 1 (applying Rule 1).
Example: Probability of at least 1 boy in family of 3 kids = 1 – Probability of all girls = 1 – (.488)^3 = 1 – .116 = .884

Finding Conditional Probability in Opposite Direction

Know P(B|A) but want P(A|B): Use Rule 3a to find P(B) = P(A and B) + P(A^c and B), then use Rule 4.

P(A | B) = \frac{P(A \text{ and } B)}{P(B | A)P(A) + P(B | A^c)P(A^c)}

Let’s look at some tools that are much easier than using this formula!
Solve these probability questions

Example 1: Medical testing for a rare disease

D = person has the disease, suppose:
P(D) = 1/1000 = .001, P(D') = .999

T = test for the disease is positive, suppose:
P(T | D) = .95, so P(T' | D) = .05
P(T | D') = .05, so P(T' | D') = .95

So the test is 95% accurate whether person has the disease or not
Find P(D | T)
= Probability of disease, given the test is positive

Two-Way Table:

“Hypothetical Hundred Thousand”
Table of hypothetical 100,000 people who get tested
1/1000 of them have disease = 100 people
Of those, 95% = 95 people test positive, so 5 negative
999/1000 of them don’t have the disease = 99,900 people
Of those, 95% = 94,905 people test negative, so 495 positive
Read from Table: P(Disease | Test positive ) = 95/5090 = .019

<table>
<thead>
<tr>
<th>Test pos</th>
<th>Test neg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disease</td>
<td>95</td>
</tr>
<tr>
<td>No disease</td>
<td>4995</td>
</tr>
<tr>
<td></td>
<td>5090</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tree Diagrams

Step 1: Determine first random circumstance in sequence, and create first set of branches for possible outcomes. Create one branch for each outcome, write probability on branch.

Step 2: Determine next random circumstance and append branches for possible outcomes to each branch in step 1. Write associated conditional probabilities on branches.

Step 3: Continue this process for as many steps as necessary.

Step 4: To determine the probability of following any particular sequence of branches, multiply the probabilities on those branches. This is an application of Rule 3a.

Step 5: To determine the probability of any collection of sequences of branches, add the individual probabilities for those sequences, as found in step 4. This is an application of Rule 2b.

Example 2: Probability of getting a B or better

Return to example of Statistics 8 grades
A = student comes to class regularly; P(A) = 0.7
B = student gets at least a B in the course

P(B|A) = 0.8     P(B|A') = 0.4

Question:
What is the overall probability of getting at least a B?

Some definitions from Section 7.7

Probability of accurate medical test

Define the events:
D = person has the disease
D' = person does not have the disease
T = test is positive
T' = test is negative

Sensitivity of a test = P(T | D), i.e., correct outcome if person has the disease.
Specificity of a test = P(T' | D'), i.e., correct outcome if person does not have the disease.

Disease probability

Sensitivity = specificity = 0.95
P(D and positive test) = (.001)(.95) = .00095.
P(test is positive) = .00095 + .04995 = .0509.
P(D | positive test) = .00095/.0509 = .019
Example 2: Return to example of Stat 8 grades
A = student comes to class regularly; P(A) = 0.7, P(A') = 0.3
B = student gets at least a B in the course
P(B|A) = 0.8  P(B|A') = 0.4
Question: What is the overall probability of getting at least a B?

```
<table>
<thead>
<tr>
<th>Class?</th>
<th>B or better</th>
<th>C or less</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>.8</td>
<td>.2</td>
</tr>
<tr>
<td>No</td>
<td>.6</td>
<td>.4</td>
</tr>
</tbody>
</table>
```

So, probability of B or better = .56 + .12 = .68 overall

Problem 1
Suppose there is no relationship between two variables, e.g. listening to Mozart and increased IQ. Suppose 3 independent experiments are done, each using the 0.05 criterion for statistical significance.

What is the probability that at least one experiment results in a statistically significant relationship just by chance?

Problem 2
Suppose you drive on a certain freeway daily. Speed limit is 65. You drive over 65 all the time, but over 75 about 30% of the time.
P(ticket | over 75) = 1/50 = .02
P(ticket | 65 to 75) = 1/200 = .005.

What is the probability you get a ticket on a randomly selected day?