Statistics 8
Sample Midterm 2 KEY

There are *6 problems on 3 pages.* Each part of each problem is worth 6 points except as shown. A copy of Table A.1 is attached. Show all work.

1. An herbal treatment is being tested to see if it will help people who are suffering from seasonal allergies. Suppose the truth is that the treatment actually will help 60% of people who take it, and results are independent from one person to the next. The treatment is administered to 3 people. Define X = number who are helped by the treatment.

   a. What is the probability that X = 0?

   Use the extension of the multiplication rule. For each person, the probability that the person won't be helped is .4. So, the probability that none of the 3 people will be helped is (.4)(.4)(.4) = .064.

   b. (6 pts=2 pts each) Define two events: **A** = the first person is helped by the treatment
   B = the second person is helped by the treatment.

Check off the answer for these questions:	Yes	No
i. Are A and B independent?	__X__	____
ii. Are A and B mutually exclusive?	____	__X__
iii. Are A and B complements?	____	__X__

   c. Event **A** is defined in the previous part as "the first person is helped by the treatment." Explain in words the event $\mathbf{A}^C$ and give the probability $P(A^C)$.

   The event A^C (the complement of A) is that the first person is not helped by the treatment. The probability is $1 - P(A) = 1 - .6 = .4$.

   d. State whether or not X is a binomial random variable. If so, specify n and p. If not, specify which condition is not met.

   Yes, X is a binomial random variable; n = 3 and p = the probability that each person will be helped by the treatment = .60.

2. (1 pt for each blank) Specify whether each of the following applies for binomial, uniform and normal random variables. Write yes or no. (Write yes only if it is true for *all* random variables of that type.)

   a. 68% of possible values are in the range $\mu \pm \sigma$.
   Binomial?__No____ Uniform?__No____ Normal?__Yes____

   b. $\mu = np$
   Binomial?__Yes____ Uniform?__No____ Normal?__No____

   c. The pdf is constant for all values in the range of possibilities.
   Binomial?__No____ Uniform?__Yes____ Normal?__No____

   d. Continuous random variable.
   Binomial?__No____ Uniform?__Yes____ Normal?__Yes____

   e. Discrete random variable
   Binomial?__Yes____ Uniform?__No____ Normal?___No___

3. Suppose that weights (X) of girls in a certain age group have a normal distribution with mean $\mu = 80$ pounds and standard deviation $\sigma = 12$ pounds. Find each of the following probabilities:
   a. $P(X \leq 92)$ = probability the weight of a randomly selected girl is less than or equal to 92 pounds.

$$P(X \leq 92) = P(Z \leq \frac{92-80}{12}) = P(Z \leq 1) = .8413 \text{ (from Table A.1)}$$

4. In a random sample of $n = 100$ students at a university, 85 said they owned their own computer. Use this information to find an approximate 95% confidence interval (*not* a conservative one) for the true proportion of students at the university who own their own computers. Show all of your work.

   The confidence interval is Sample estimate ± Multiplier x Standard error

   Sample estimate = $\hat{p} = .85$*; Multiplier = 2.0; Standard error =* $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(.85)(.15)}{100}} = .0357$

   The confidence interval is .85 ± (2.0)(.0357) or .85 ± .0714 or .7786 to .9214.

5. Matt has been hired to be a telemarketer. He must complete 100 calls a day in which someone actually talks with him. Therefore, during his first week on the job he must complete 500 calls. The company expects him to be successful at least 15% of the time, so during the first week they expect him to be successful in at least 75 calls (15% of 500). Define X = number of calls out of 500 that Matt is successful. Assuming his success is independent from one call to the next, X is a binomial random variable.

   a. If in fact the probability that Matt will be successful on each call is .18, find the mean and standard deviation of X.

      Because X is binomial, mean = np = (500)(.18) = 90
      standard deviation = $\sqrt{np(1-p)} = \sqrt{500(.18)(.82)} = \sqrt{73.8} = 8.59$

   b. (10 pts) Again assuming the probability that Matt will be successful on each call is .18, use the normal approximation to the binomial to find the probability that Matt will *not* meet his quota of 75 calls in the first week. In other words, find (approximately) $P(X \leq 74)$.

$$P(X \leq 74) \approx P(Z \leq \frac{74-90}{8.59}) = P(Z \leq -1.86) = .0314 \text{ (from Table A.1)}$$

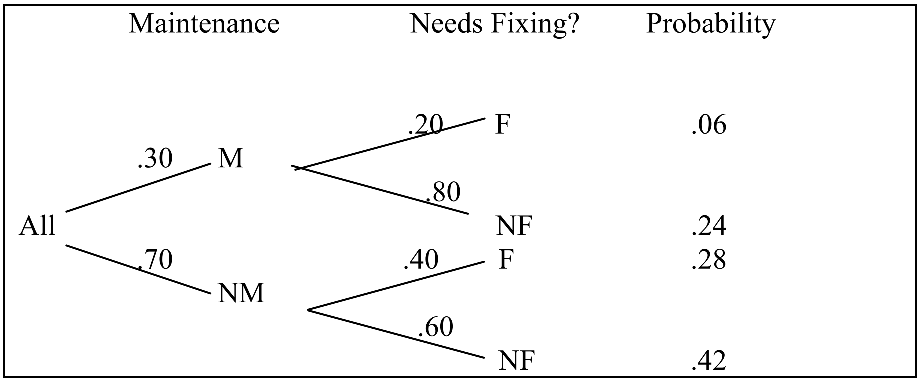
   c. (10 pts) Define $\hat{p}$ to be the *proportion* of successful calls Matt has out of 500 calls he makes. Again assume the probability that he is successful on each call is .18. Describe the sampling distribution of $\hat{p}$, including numerical values for its mean and standard deviation.

      The sampling distribution of $\hat{p}$ *is approximately normal with mean = p = .18*

      and standard deviation = s.d.($\hat{p}$ *) =* $\sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(.18)(.82)}{500}} = .017$

6. An automobile company has noticed that 30% of people who buy a new car bring it in regularly to the dealer for maintenance. For people who do get regular maintenance, the probability is .20 that they will need something fixed during the warranty period. For people who don't get regular maintenance, the probability is .40 that they will need something fixed during the warranty period.

   a. (10 pts) Draw a tree diagram for this situation ***or*** construct a "hypothetical hundred thousand" table. (To save time, you can use the abbreviations M = regular maintenance and NM = no regular maintenance; F = need something fixed and NF = no need to have something fixed during warranty.)



Hypothetical hundred thousand table:

	F	NF	Total
M	6,000	24,000	30,000
NM	28,000	42,000	70,000
Total	34,000	66,000	100,000

   b. What is the overall probability that someone who buys one of these cars will need something fixed during the warranty period? (Your results from Part (a) should help you to answer this.)

From the tree diagram, it's the sum of the probabilities for the branches ending in "F" = .06+.28=.34.

From the hypothetical hundred thousand table, it's the proportion in the column "F" $= \dfrac{34{,}000}{100{,}000} = .34.$

   c. Given that someone needs something fixed during the warranty period, what is the probability that he or she did *not* get regular maintenance?

From the tree diagram and the probability rules, it is P(NM | F) $= \dfrac{P(NM \text{ and } F)}{P(F)} = \dfrac{.28}{.34} = .8235$

From the table, it's the proportion of NM in the F column $= \dfrac{28{,}000}{34{,}000} = .8231$

7. (1 pt) Which of the following sequences resulting from tossing a fair coin 5 times is *most* likely, where H=head and T=tail? (Circle your choice):

   HHHHH
   HTHHT
   HHHTT
   They are equally likely

   All sequences are equally likely because P(Heads) = P(Tails) = .5. The probability for each sequence is $(1/2)^5$ *= 1/32 by the multiplication rule for independent events.*