Preparation of Two-Year College Mathematics Instructors to Teach Statistics with GAISE

Session on Teaching with Active Learning

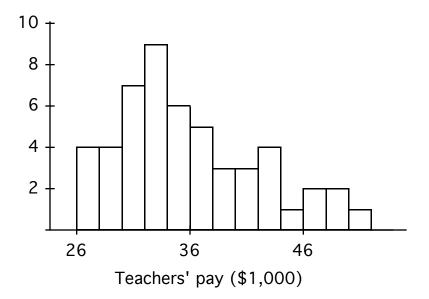
Sorting Distributions into Shapes

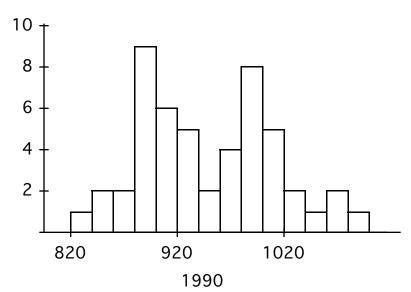
- (5 10 minutes) On the next five pages are histograms that represent the distributions for different types of measurements. Your task is to look at the shape of each histogram and then sort the graphs into piles so that the graphs in each pile have a similar shape.
- 1. Get into a group of 3 or 4 people.
- 2. Tear out the next five pages of graphs from one of your Course Manuals.
- 3. Cut apart the graphs so there is one graph on each sheet of paper.
- 4. Work with each other in the group to create piles of graphs that have the same shape.
- (5 10 minutes) Once you are satisfied, as a group, with the placement of the graphs into piles, do the following for each pile of graphs.
- 1. Select one graph from each pile that you think is the most typical example of graphs in a pile.
- 2. Decide on a word or phrase that can be used to label each pile. The name should capture some feature that is typical of all the graphs in a pile.
- 3. Write a short description of two or three characteristics that define the graphs in each pile. See if you can come up with characteristics that help you distinguish the graphs in one pile from another pile.

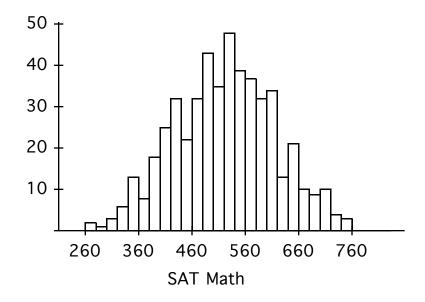
Once each group has finished with their descriptions, we will see what each group has come up with compare similarities and differences.

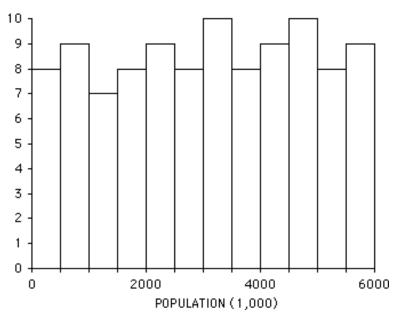
The following URL provides a page that can be used to provide feedback to students and facilitate discussion:

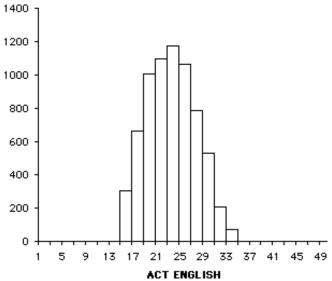
http://ore.gen.umn.edu/faculty_staff/delmas/gc_1454_course/distribution_files/distribution.html

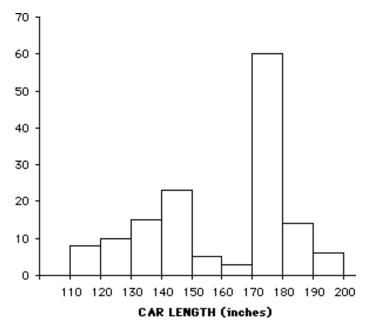


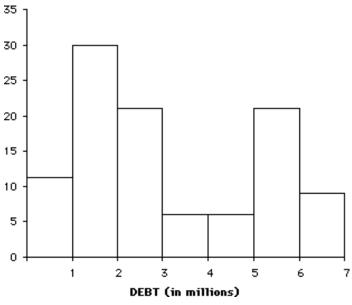


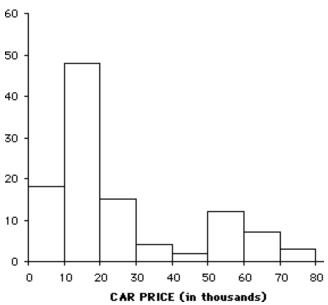


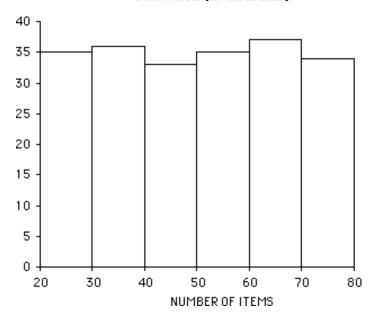


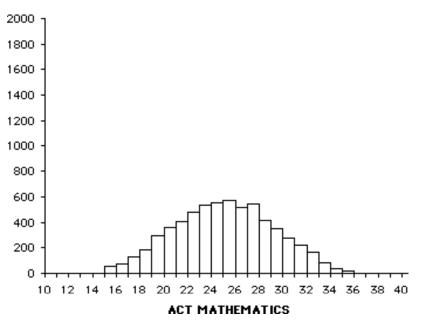


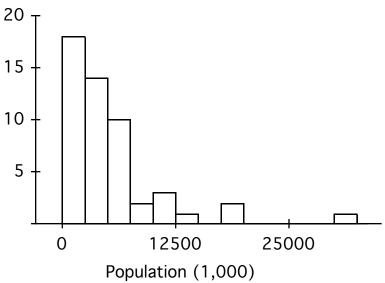


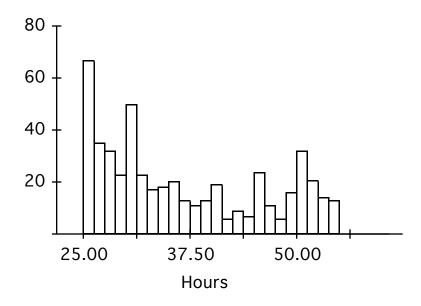


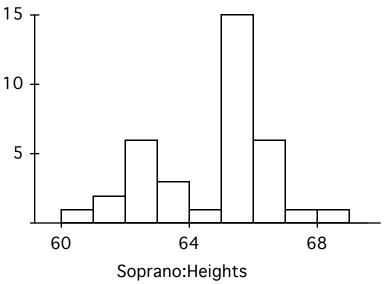


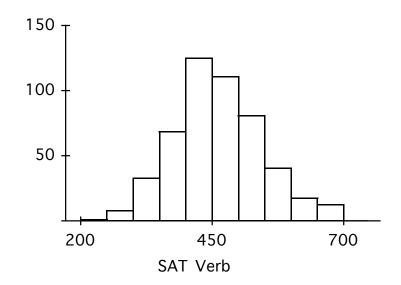


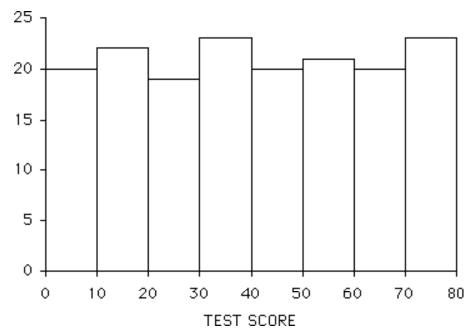








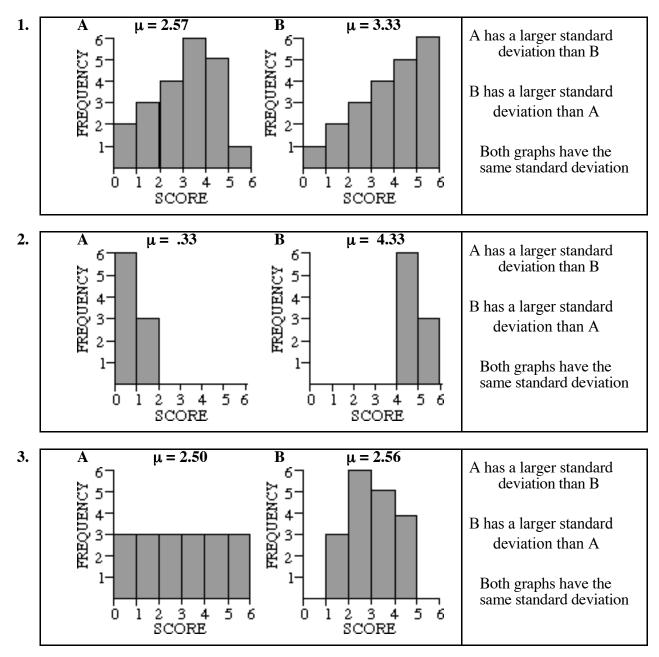




What Makes the Standard Deviation Larger or Smaller?

There are three graphs on this page. After you complete this page, you will receive another page with three graphs. You are given the mean (μ) for each graph. For each pair of graphs:

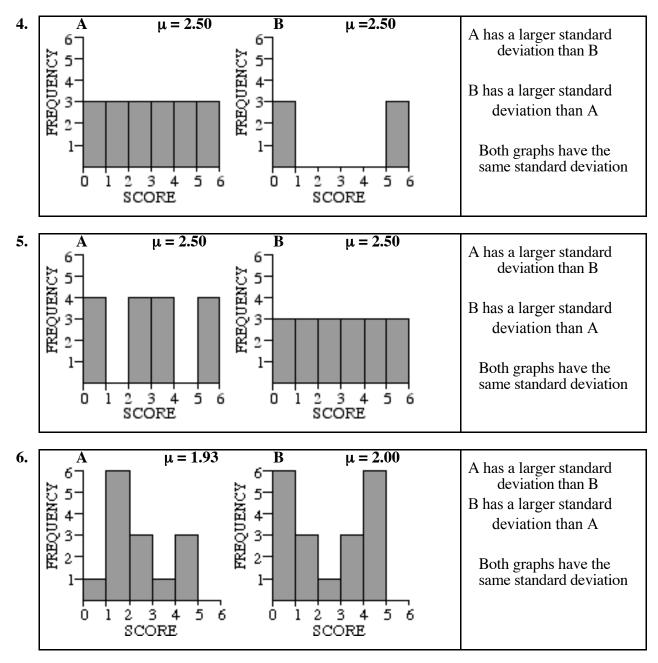
- a. Use the **FACTORS SHEET** to identify the characteristics of the graphs that might make the standard deviation larger or smaller.
- b. On this sheet and on the FACTORS SHEET, indicate whether one of the graphs has a LARGER standard deviation than the other or if the two graphs have the SAME standard deviation.
- c. Check your answers with the answer key at the course homepage after you complete the page: http://ore.gen.umn.edu/faculty_staff/delmas/gc_1454_course/variability_test/var_answers.html



What Makes the Standard Deviation Larger or Smaller?

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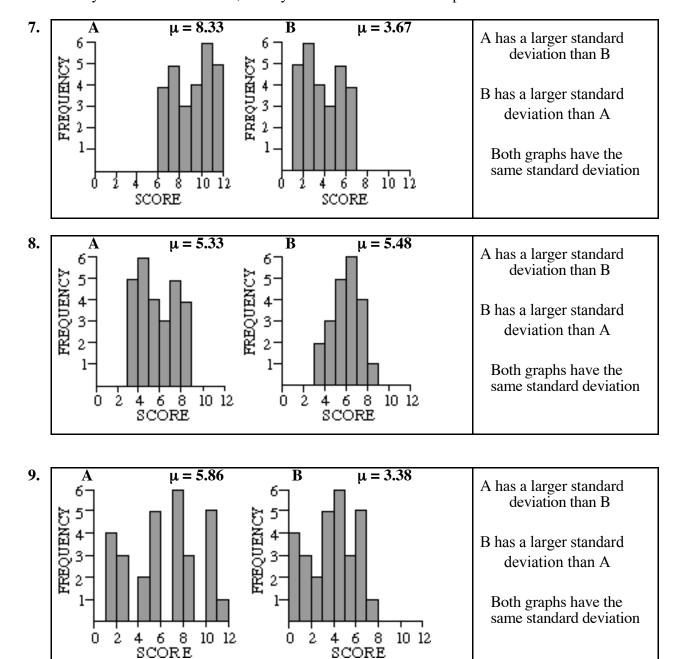
- a. Use the **FACTORS SHEET** to identify the characteristics of the graphs that might make the standard deviation larger or smaller.
- b. On this sheet and on the FACTORS SHEET, indicate whether one of the graphs has a LARGER standard deviation than the other or if the two graphs have the SAME standard deviation.
- c. Check your answers with the answer key at the course homepage after you complete the page: http://ore.gen.umn.edu/faculty_staff/delmas/gc_1454_course/variability_test/var_answers.html



What Makes the Standard Deviation Larger or Smaller?

This is your last sheet of graphs. For each pair of graphs:

- a. Use the **FACTORS SHEET** to identify the characteristics of the graphs that might make the standard deviation larger or smaller.
- b. On this sheet and on the FACTORS SHEET, indicate whether one of the graphs has a LARGER standard deviation than the other or if the two graphs have the SAME standard deviation.
- c. Check your answers with the answer key at the course homepage after you complete the page: http://ore.gen.umn.edu/faculty_staff/delmas/gc_1454_course/variability_test/var_answers.html Given what you have learned so far, see if you can make three correct predictions.

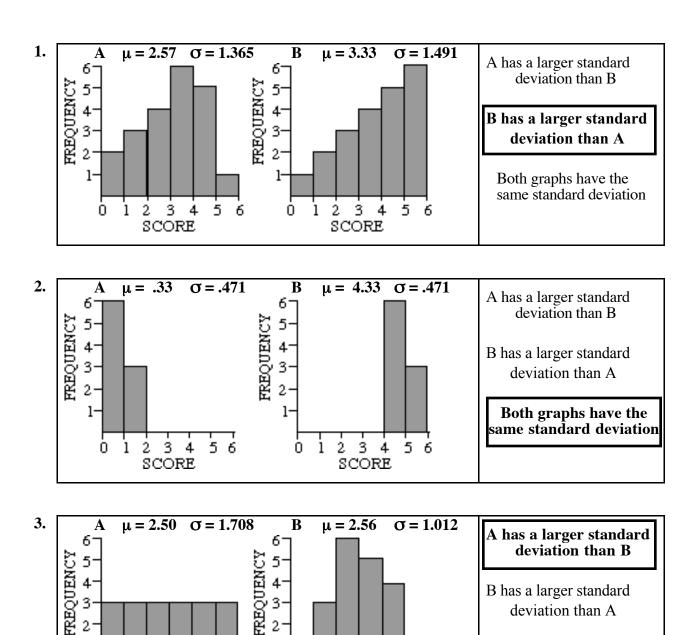


FACTORS SHEET NAME:		PROBLEM							
		SHEET 1		SHEET 2		SHEET 3			
		1	2	3	4	5 6	7	8	9
RANGE:	Which distribution has the wider range (the difference between the Highest and Lowest values)?	A B SAME	A B SAME	A B SAME		A B A B SAME	A B SAME	A B SAME	A B SAME
BUMPINESS:	In which distribution do the bar heights go up and down the most?	A B SAME	A B SAME	A B SAME		A B A B SAME	A B SAME	A B SAME	A B SAME
HIGH DENSITY:	Which distribution has a higher percentage of values concentrated around the mean (µ)?	A B SAME	A B SAME	A B SAME		A B A B SAME	A B SAME	A B SAME	A B SAME
SPREAD AWAY FROM MEAN:	Which distribution has a higher percentage of values spread away from the mean (µ)?	A B SAME	A B SAME	A B SAME		A B A B SAME	A B SAME	A B SAME	A B SAME
HIGHER MEAN:	Which distribution has the higher mean?	A B SAME	A B SAME	A B SAME		A B A B SAME	A B SAME	A B SAME	A B SAME
MORE BARS:	Which distribution has a larger number of bars?	A B SAME	A B SAME	A B SAME		A B A B SAME	A B SAME	A B SAME	A B SAME
	RIBUTION DO YOU PREDICT RGER STANDARD DEVIATION?	A B SAME	A B SAME	A B SAME		A B A B SAME	A B SAME	A B SAME	A B SAME
CHECK YOUR ANSWER:	Which graph ACTUALLY has the larger standard deviation?	A B SAME	A B SAME	A B SAME		A B A B SAME	A B SAME	A B SAME	A B SAME

Answers to the items for What Makes the Standard Deviation Larger or Smaller:

ITEM	sd of Graph A	sd of Graph B	Correct Decision
1	$\sigma = 1.365$	$\sigma = 1.491$	B is larger
2	$\sigma = .471$	$\sigma = .471$	SAME
3	$\sigma = 1.708$	$\sigma = 1.012$	A is larger
4	$\sigma = 1.708$	$\sigma = 2.50$	B is larger
5	$\sigma = 1.803$	$\sigma = 1.708$	A is larger
6	$\sigma = 1.280$	$\sigma = 1.686$	B is larger
7	$\sigma = 1.743$	$\sigma = 1.743$	SAME
8	$\sigma = 1.743$	$\sigma = 1.332$	A is larger
9	$\sigma = 3.126$	$\sigma = 2.075$	A is larger

ANSWERS TO PROBLEMS 1, 2, AND 3



3

SCORE

4

2

5

6

Both graphs have the same standard deviation

1-

Ò

1-

Ò

3

SCORE

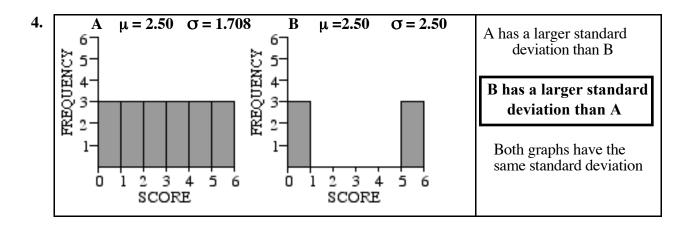
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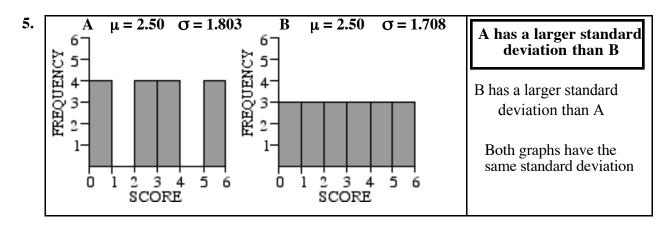
2

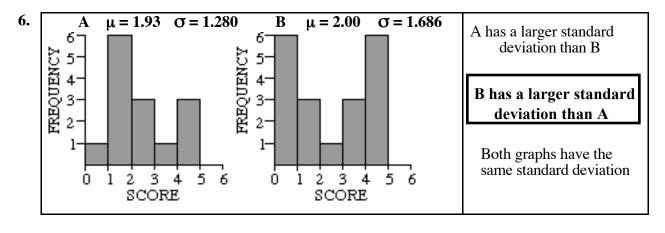
5

6

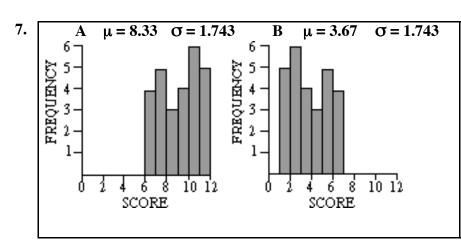
ANSWERS TO PROBLEMS 4, 5, AND 6







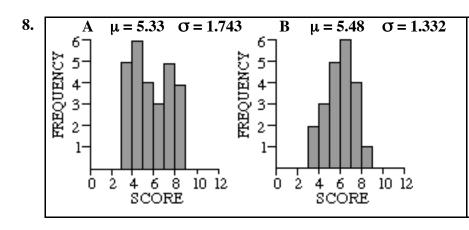
ANSWERS TO PROBLEMS 7, 8, AND 9



A has a larger standard deviation than B

B has a larger standard deviation than A

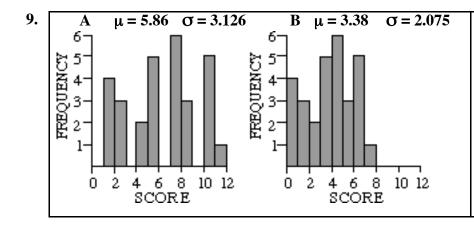
Both graphs have the same standard deviation



A has a larger standard deviation than B

B has a larger standard deviation than A

Both graphs have the same standard deviation



A has a larger standard deviation than B

B has a larger standard deviation than A

Both graphs have the same standard deviation

SAMPLING DISTRIBUTIONS ACTIVITY Part 1

The Situation: Joe, Talia, Cally, Desmond, and Nick are five clerks who work in the small claims office for the government of a large metropolitan city. Their supervisor periodically checks up on them to see how long it takes each of them to process claims. In order to deal with the large volume of small claims that come through the office each day, each clerk must take 6 minutes or less to process a claim, on the average. It would be too time consuming, if not impossible, for the supervisor to monitor every transaction performed by each clerk. The supervisor needs to decide on the best way to make a reliable estimate of the time it takes each clerk to process claims. In general, she has one of three options:

Small Sample: Pick a **small** random sample (somewhere between 2 and 4 claims).

Medium Sample: Pick a **medium** random sample (somewhere between 8 and 12 claims).

Large Sample: Pick a large random sample (somewhere between 15 and 20 claims).

The supervisor would choose only one sample. For any sample she chooses, she would calculate the average time it took the employee to process the claims in the sample. If a clerk takes longer than 6 minutes, on the average, to process small claims, the clerk may be moved to a less demanding position, or possibly fired. The supervisor does not want to make an incorrect decision and dismiss a clerk who is actually performing their job well. Your task is to find the method that will help the supervisor make the best decision.

<u>Prediction</u>: Which monitoring method should the supervisor choose?

Small Sample Medium Sample Large Sample Doesn't Matter

Your instructor will now lead you through several tasks that look at what might happen if the supervisor used each of the three methods with different employees. This activity uses a piece of software called **Sampling SIM**. To download a free copy of Sampling SIM, go to the following URL:

http://www.gen.umn.edu/research/stat_tools/

Then click the **SOFTWARE** button on the left side of the web page.

First Employee: Joe

Joe's claim processing times follow a Normal Distribution

Second Employee: Talia

Talia's claim processing times do NOT follow a Normal Distribution

Third Employee: Cally

Cally's claim processing times have a very Erratic (Irregular) pattern

Question 1	Look at the graphs in the first row of the Scrapbook when the sample For which populations does most of the class say that the SHAPE of the sample means was About the Same or only A Little Different when shape of the population?				distribution of
	NORMAI	L NE	GATIVE SKEW	IRREGULA	AR
Question 2 Look at the graphs in the second and third rows of the Scrapbook when the sizes were n = 9 and n = 16 . Is there a population for which most of the classical the SHAPE of the distribution of sample means was About the Same or or Different when compared to the shape of the population?					class agrees that
	NORMAI	L NE	GATIVE SKEW	IRREGULA	AR
Question 3	Look again at the graphs in the second and third rows of the Scrapbook when the sample sizes were $n = 9$ and $n = 16$. For which populations does most of the class so the distribution of sample means has a Different or Very Different shape compared the shape of the population?				
	NORMAI	L NE	GATIVE SKEW	IRREGULA	AR
Question 4	How would you shape that is diffe		pe of those distribut opulation?	ions of sample mear	ns that have a
	medium (around	n = 9) or larger	nple means have this ? In other words, what causes the distrib	hat is it about the pr	ocess of creating
Question 5	these mean value	s compare to the	ded for the Mean of e population mean? st compare the value arm of graphs.	Remember that ther	e are three
	Much Lower	A Bit Lower	About the Same	A Bit Higher	Much Higher
Question 6		Again, what is it	pple means always he about the process o		

Sampling Distributions Activity – Part 1

Question 7	Next, consider the values that you recorded for the value of the sd of \overline{x} for each of the graphs. In general, what happened to the variability of the distribution of sample means (what happened to the size of sd of \overline{x}) as sample size increased from $n=2$ to $n=9$ to $n=16$?
Question 8	Which sample size always produced the LARGEST variability among sample means? 2 9 16
Question 9	Which sample size always produced the SMALLEST variability among sample means? 2 9 16
Question 10	Why does the variability in the distribution of sample means decrease when the sample size is increased? What is it about a larger sample size that causes the variability of the sample means to decrease?
Question 11	Again, look at all the values that you recorded for sd of \overline{x} for all of the graphs. Was it ever the case that the sd of \overline{x} was LARGER than the standard deviation of the population (the value of σ)? YES NO
	Explain why this is true.

Question 12	Let's go back to the original problem of deciding which monitoring method the supervisor should use. To make this decision we'll look one more time at the distribution of sample means for each sample size. Remember that the supervisor will take only ONE sample of processing times for each employee. The supervisor wants to make the right decision: keep employees who have fast processing times, and fire employees who take too long. The question is which sample size should the supervisor use to make her decision, small, medium, or large?							
	A) First, look at the very top of the scrapbook page and locate the population means (μ) for Joe, Talia, and Cally. Based on this information, which employees have an average processing time that is more than 6 minutes and, therefore, should be fired?							
		Joe	Talia	Cally				
	Which employees have an average processing time that is less than 6 minutes and, therefore, should NOT be fired?							
		Joe	Talia	Cally				
	Next, let's look at the chances of the supervisor correctly deciding who should be fired and who should be retained based on each sample size.							
	B) Look at the distribution of means for the small samples $(n = 2)$. First, estimate what percent of the distribution appears to be greater than 6 for each employee. Each of these percents represents the probability of obtaining a sample mean for a sample of size 2 that is greater than 6. This is an estimate of the probability that each employee will be fired if only observed twice $(n = 2)$ by the supervisor.							
	Joe	Ta	ılia	Cally				
	C) Look at the distribution of means for the medium sized samples $(n = 9)$. Estimate the probability that each employee will be fired if observed nine times $(n = 9)$ by the supervisor.							
	Joe	Ta	alia	Cally				
	D) Look at the distribution of means for the large samples $(n = 16)$. Estimate the probability that each employee will be fired if observed sixteen times $(n = 16)$ by the supervisor.							
	Joe	Ta	alia	Cally				
	E) Which sample size gives the GREATEST chance of CORRECTLY firing the employee(s) whose mean processing time is MORE than 6? 2 9 16							
	F) Which sample size gives the LOWEST chance of INCORRECTLY firing the employee(s) whose mean processing time is LESS than 6? 2 9 16							
	Given the supervisor's goal to make correct decisions about firing and retaining the employees, as well as the practical considerations in terms of time to observe the employees, what do you think is the best advice to give the supervisor? Write a memo to the supervisor that recommends the best sample size to use and state why this is the best sample size. Remember that the supervisor has not participated in this activity and may not have an understanding of statistics.							

Sampling Distributions Activity Part 2

The purpose of Part 2 is to test your understanding of how sampling distributions behave and to learn additional details about sampling distributions. First, let's see if you can used what you have learned about distributions of sampling means.

You will consider three populations. One of the populations is Joe's Normal Distribution with which you're already familiar. The other two populations are for the other two employees, Desmond and Nick. Desmond has a population that can be described as trimodal, and Nick's population has an irregular shape with four modes.

Imagine that the supervisor decides to draw a sample of size n = 30 to get an accurate appraisal of each employee's claim processing times.

A Return to Case 1: Joe's Normal Distribution

1. Turn to page 2 of the Sampling Distributions Scrapbook. At the top of the first column you will see a picture of Joe's Normal Distribution. Look at the second sheet of graph labels. The first column contains possible distribution of sample means from 500 samples, each of size n = 30..

From the first column, locate the graph that you think represents a distribution of means for 500 samples, each of size n = 30. Go to the page 2 of the Sampling Distribution Scrapbook and CIRCLE the letter (A, B, C, D, or E) after the words **Guess 10**: in the box for **Distribution of Sample Means** n = 30.

Case 4: Desmond's Trimodal Distribution

2. On page 2 of the Sampling Distributions Scrapbook you will see a picture of Desmond's trimodal population at the top of the second column. Look at the second sheet of graph labels. The second column contains possible distributions for 500 sample means from samples of size n = 30 taken from Desmond's population.

From the second column, locate the graph that you think represents a distribution of means for 500 samples, each of size n = 30. Go to the page 2 of the Sampling Distribution Scrapbook and CIRCLE the letter (A, B, C, D, or E) after the words **Guess 11:** in the box for **Distribution of Sample Means** n = 30.

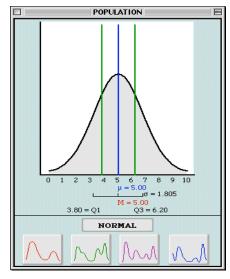
Case 5: Nick's Four-Mode Distribution

3. For your third guess, look at the top of the third column of the Sampling Distributions Scrapbook that presents Nick's irregular, four-mode population.

From the third column of the second sheet of graph labels, locate the graph that you think represents a distribution of means for 500 samples, each of size n = 30. Go to the page 2 of the Sampling Distribution Scrapbook and CIRCLE the letter (A, B, C, D, or E) after the words **Guess 12:** in the box for **Distribution of Sample Means** n = 30.

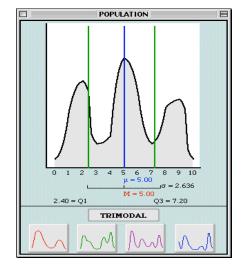
4. Checking Your Guess for Case 1: Joe's Normal Distribution with n = 30

CLEAR DISTRIBUTION
BIMODAL
NORMAL
SKEWED +
SKEWED -
TRIMODAL
U-SHAPED
UNIFORM

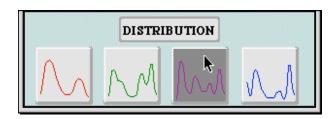


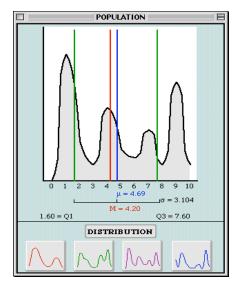
5. Checking Your Guess for Case 4: Desmond's Trimodal Distribution with n = 30

CLEAR DISTRIBUTION
BIMODAL
NORMAL
SKEWED +
SKEWED TRIMODAL
U-SHAPED
UNIFORM



6. Checking Your Guess for Case 5: Nick's Four-Mode Distribution with n = 30





Sampling Distributions Activity – Part 2

The next section introduces some ideas from an important theorem called the **Central Limit Theorem**.

What are three implications of the Central Limit Theorem?

1.

2.

3.

Recall the 68-95-99.7 rule for a Normal Distribution.

What percent of the values will be between 1 standard deviation below the mean and 1 standard deviation above the mean?

%

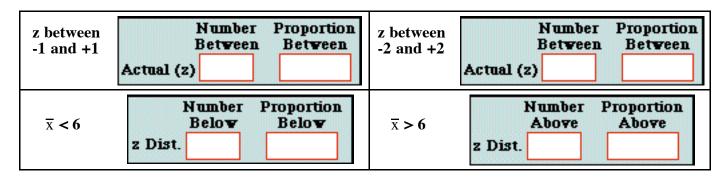
What percent of the values will be between 2 standard deviations below the mean and 2 standard deviations above the mean?

______9

The Central Limit Theorem doesn't just say that a sampling distribution will LOOK like a Normal Distribution.

The Central Limit Theorem claims that the sampling distribution will BEHAVE like a Normal Distribution, given that the size of each sample is large enough (again, n = 30 or larger).

Sampling Distributions from a Normal Distribution: Case 1, Joe's Normal Distribution



Is it likely that Joe will be moved to another position or fired?

YES

NO

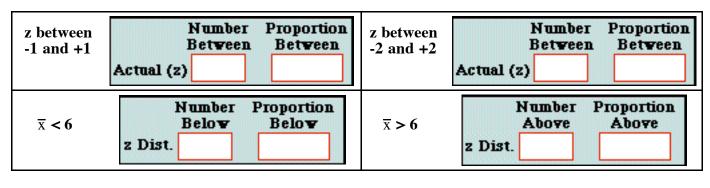
Sampling Distributions from Populations that Are Not Normal Distributions

When the population is a Normal Distribution, we are guaranteed that the sampling distribution will also be a Normal Distribution, exactly. What happens when the population is not a Normal Distribution? The Central Limit Theorem states that when the sample size is large enough, the sampling distribution will be approximated quite well by a Normal Distribution. Statisticians have found that a sample size of 30 or larger is usually large enough for this statement to be true. Let's see if this is the case for Nick since his population is definitely not normal.

Sampling Distribution for Case 6: Nick's Four-Mode Population

z between -1 and +1	Number Proportion Between Between Actual (z)	z between -2 and +2	Number Proportion Between Between Actual (z)		
₹ < 6	Number Proportion Below Below z Dist.	₹ > 6	Number Proportion Above Above z Dist.		
Is it likely that Nick will be moved to another position or fired? YES NO					

Sampling Distribution for Case 2: Talia's Negatively Skewed Population



Is it likely that Talia will be moved to another position or fired? YES NO

Now, answer the questions on pages 5 and 6 to summarize what you have learned.

3. Joe's population of claim-processing times formed a normal distribution, with mean (μ) of 5 and a standard deviation $\sigma = 1.805$. Use the 68-95-99.7% rule to estimate the times that include the middle 68% of Joe's distribution of times.

Sampling Distributions Activity – Part 2

4. Now, imagine a sampling distribution of all samples size n=4 from Joe's population of claim-processing times. Remember the mean (μ) is still 5, but now the standard deviation is measured by $\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$, which is $\frac{\sigma}{\sqrt{n}} = \frac{1.805}{\sqrt{4}}$. This number is about 0.9. Use this number instead of sigma and apply the 68-95-99.7% rule to the SAMPLING DISTRIBUTION to estimate the sample mean claim-processing times (from samples size n=4) that would include the middle 68% of the distribution of sample means.

Now, repeat to estimate the middle 95% of the distribution.

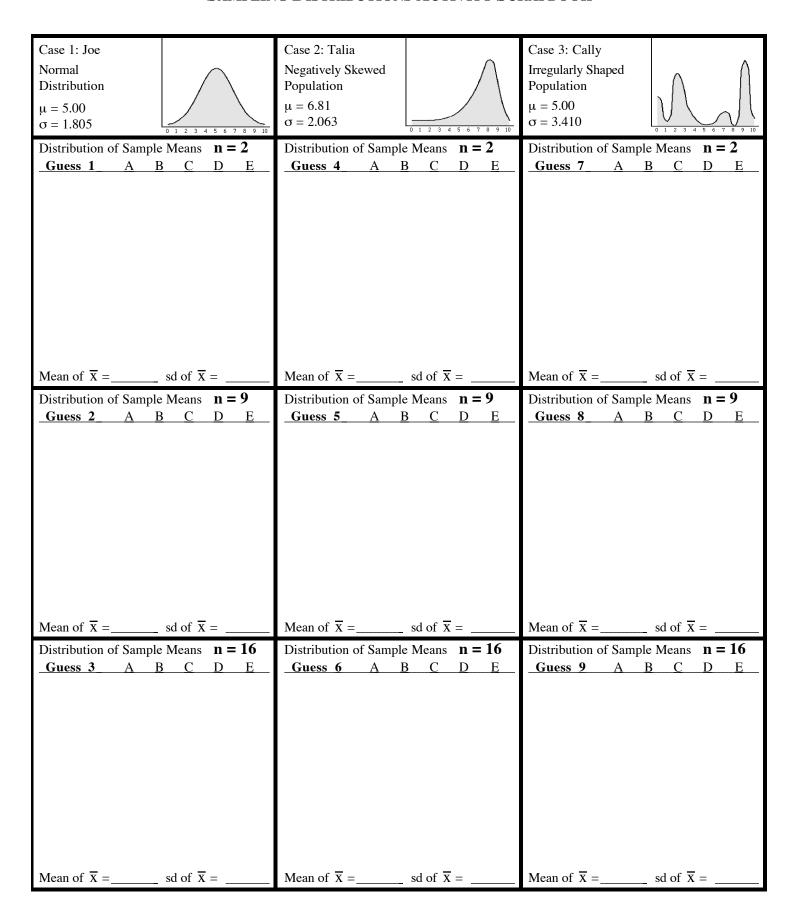
If we had taken samples of size 25 instead of size 4, what sample mean claim-processing times would include the middle 68% of this sampling distribution?

What about the middle 95%?

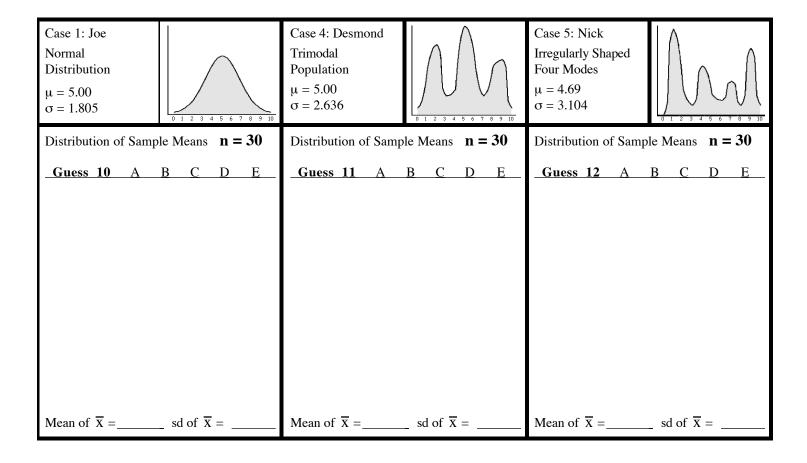
5. Next we'll consider Talia's skewed distribution of claim-processing times and repeat the same exercise. We now know that if we take large samples from a skewed distribution, the sampling distribution should look approximately normal. So, although we CAN'T apply the 68-95-99,7% rule to her POPULATION of claim-processing times, we CAN apply it to a SAMPLING DISTRIBUTION of sample means if the sample size is large.

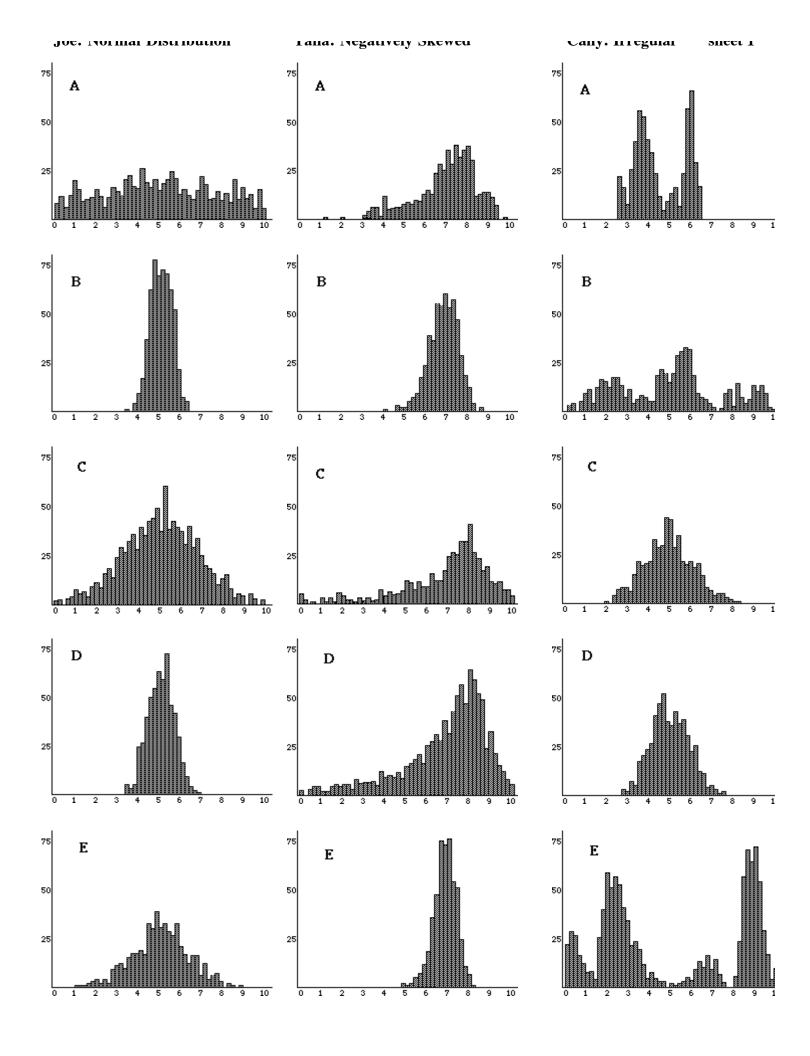
Think about a sampling distribution from her population (mean $\mu=6.81$ and standard deviation $\sigma=2.063$). If we were to draw large samples of size n=50, then what sample mean claim-processing times would include the middle 68% of her sampling distribution? Remember to first compute the value of $\sigma_{\overline{x}}=\frac{\sigma}{\sqrt{n}}$, then apply the 68-95-99.7% rule to find the middle 68%.

SAMPLING DISTRIBUTIONS ACTIVITY SCRAPBOOK



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pling distributions scrapbook with pasted graphs and illustrative summary statistics.

