ICS 271  
Fall 2015  
Instructor : Kalev Kask  
Homework Assignment 5  
Due Tuesday November 10

1. (15) Consider a vocabulary with only four propositions, $A$, $B$, $C$ and $D$. How many models are there for the following sentences:

(a) $A \lor B$
(b) $(A \Rightarrow B) \land A \land \neg B \land C \land D$
(c) $\neg A \lor \neg B \lor \neg C \lor \neg D$

2. (15) Consider the statement “The car is either at John’s house or at Fred’s house. If the car is not at John’s then it must be at Fred’s house.”

(a) Describe a set of propositional letters which can be used to represent these statements.
(b) Describe the statements using a propositional formula on the propositions you described for (a).
(c) Can you determine where is the car?

3. (10) How would you use the truth table to prove that modus ponens is sound.

4. (10) Convert the following propositional calculus wff into CNF form:

$$\neg[(R \lor \neg Q) \rightarrow P) \rightarrow (R \land Q)]$$

5. (10) Show how the N-Queens problem can be represented as a Propositional Satisfiability (PSAT) problem. (Hint: Introduce a proposition $q_{k,l}$ for each square $(k,l)$ of the $N \times N$ board. If $q_{k,l}$ has value True, there is a queen on square $(k,l)$; if it has value False, that square is empty. Now state the constraints of the problem in terms of these propositional symbols.)

6. (20) Use truth tables to show that the following sentences are valid, and thus that the equivalences hold. Some of these equivalence rules have standard names, which are given in the right column.

$P \land (Q \land R) \Leftrightarrow (P \land Q) \land R$  
$P \land (Q \lor R) \Leftrightarrow (P \land Q) \lor (P \land R)$  
$\neg(P \land Q) \Leftrightarrow \neg P \lor \neg Q$  
$P \Leftrightarrow Q \Leftrightarrow (P \land Q) \lor (\neg P \land \neg Q)$

Associativity of conjunction  
Associativity of conjunction  
de Morgan’s Law  
Associativity of conjunction

7. (30) Look at the following sentences and decide for each if it is valid, unsatisfiable, or neither. Verify your decisions using truth tables, or by using the equivalences.
(a) \(\text{Smoke} \Rightarrow \text{Smoke}\)
(b) \(\text{Smoke} \Rightarrow \text{Fire}\)
(c) \((\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow (\neg \text{Smoke} \Rightarrow \neg \text{Fire})\)
(d) \((\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow ((\text{Smoke} \land \text{Heat}) \Rightarrow \text{Fire})\)
(e) \(((\text{Smoke} \land \text{Heat}) \Rightarrow \text{Fire}) \iff ((\text{Smoke} \Rightarrow \text{Fire}) \lor (\text{Heat} \Rightarrow \text{Fire}))\)
(f) \((\text{Big} \land \text{Dumb}) \lor \neg \text{Dumb}\)

8. (15) Trace the behavior of DPLL on the knowledge-base in Figure 7.16 when trying to prove \(Q\), and compare this behavior with that of forward chaining algorithm.

9. (Extra credit) (20) The problem of finding a model to a propositional formula in conjunctive normal form (CNF) is called \textit{Propositional satisfiability (PSAT)} and is known to be NP-complete. However the problem is polynomial when the set of clauses are Horn clauses.

- Show that unit resolution (resolution when one of the clauses is a literal) is refutation complete for Horn clauses.
- Analyze the complexity of applying unit resolution to Horn clauses. Can you provide an ordering on unit-resolution so that the algorithm will terminate in polynomial time? in linear time?