Equation of a line:

Explicit: \( y = mx + c \)

How to find this given two points?

\[ \text{A} = (5, 8), \quad \text{B} = (9, 11) \]

\[ 8 = 5m + c \quad \text{--- (1)} \]
\[ 11 = 9m + c \quad \text{--- (2)} \]

\((2) - (1)\) gives

\[ 4m = 3 \]
\[ \therefore m = \frac{3}{4} \]

Plugging \( m = \frac{3}{4} \) in (1)

\[ 8 = 5 \cdot \frac{3}{4} + c \]
\[ \therefore c = 17\frac{1}{4} \]

\[ \therefore y = \frac{3}{4}x + 17\frac{1}{4}, \quad 3x - 4y + 17 = 0 \]

(implicit)
Case I

\[ 0 \leq m \leq 1 \implies \text{Angle} \leq 45^\circ \]

Let us consider two pts \((x_1, y_1)\) and \((x_2, y_2)\) lying on the line.

\[ y_2 = mx_2 + c \]
\[ y_1 = mx_1 + c \]

\[ \therefore \frac{y_2 - y_1}{x_2 - x_1} = m = \frac{\Delta y}{\Delta x} \quad \text{(3)} \]

DDA method essentially calculates \( \Delta y \) for \( \Delta n = 1 \).

\[ \therefore \text{Everytime we step 1 unit in } x\text{ direction, how much should we advance in } y\text{ direction.} \]

From (3) we know that if \( \Delta n = 1 \)

\[ \therefore \Delta y = m \Delta x \]
Algorithm

\[ A = (5, 8) \quad B = (9, 11) \]
Mark \( A \), \( n = A_x \), \( y = A_y \)
\[ n = n + \Delta x \]
\[ y = y + m \]
Mark \((n, \text{round}(y))\)
Do this till the \( y \) of \( B \) is reached.
* Pixels accessed by their bottom left corner

\[ \begin{array}{cccccc}
12 & 11 & 10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \\
\hline
12 & 11 & 10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \\
\end{array} \]

**Step 1**
\[ n = 5 \], \( n + 1 = 6 \)
\[ y = 8 + \frac{3}{4} = 8.75 \]
Mark \((6, 9)\)

**Step 2**
\[ n = 7 \]
\[ y = 8.75 + 0.75 = 9.5 \]
Mark \((7, 9)\)

**Step 3**
\[ n = 8 \]
\[ y = 9.5 + 0.75 = 10.25 \]
Mark \((8, 10)\)

**Step 4**
\[ n = 9 \]
\[ y = 10.25 + 0.75 = 11 \]
Mark \((9, 11)\)
$n = 9$, $j = 11$

reached $B$ & hence algorithm terminates.

Problems

what if $m > 1$.

Say $A = (5, 8)$, $B = (8, 12)$

$\therefore 8 = 5m + c$

$12 = 8m + c$

$\therefore m = \frac{4}{3} = 1.33$

\[\begin{array}{|c|c|c|}
\hline
x & y & \text{mark (x, y)} \\
\hline
5 & 6 & \text{mark (5, 6)} \\
6 & 7 & \text{mark (6, 7)} \\
7 & 8 & \text{mark (7, 8)} \\
8 & 9 & \text{mark (8, 9)} \\
9 & 10 & \text{mark (9, 10)} \\
\hline
\end{array}\]

Break or gap in the line.

\[\begin{align*}
\text{Step 1} & \quad n = 6 \\
& \quad y = \frac{8 + y}{2} = 9.33 \\
& \quad \text{mark (6, 9.33)} \\
\text{Step 2} & \quad n = 7 \\
& \quad y = 9.33 - \frac{y}{3} = 10.66 \\
& \quad \text{mark (7, 10.66)} \\
\text{Step 3} & \quad n = 8 \\
& \quad y = 10.66 - \frac{y}{3} = 12 \\
& \quad \text{mark (8, 12)} \\
\end{align*}\]
What are acceptable lines?

a) No gaps between adjacent pixels → CONTINUOUS
b) Pixels close to ideal line → ACCURATE
c) Smooth looking → AESTHETIC
d) Even brightness in all orientations → UNIFORMITY IN THICKNESS
e) Same line for AB & BA → CONSISTENT.

How to handle different m in DDA?

Case I \( m > 1 \)

a) Flip line about \( y=x \)
   b) Achieved by flipping \( x \) & \( y \) coordinates
b) Rasterize
c) Flip it back
   c) Switch \( x \) & \( y \) coordinates again

Case II \(-1 \leq m \leq 1\)

a) Flip about \( x \) axis
   b) Negate \( y \)
b) Rasterize
c) Flip it back
Disadvantages of DDA

Rounding & finding \( y \) are floating point operations, hence very expensive, especially since it needs to be done for every pixel.

Bresenham Method (Using Integer Arithmetic only)

Again, consider \( 0 \leq m \leq 1 \).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{dy}{dx}
\]

\[
\therefore \quad y = mx + c
\]

\[
= \frac{dy}{dx} \cdot x + c
\]

\[
\therefore \quad dy = dy \cdot x + dx \cdot c
\]

\[
\therefore \quad dy \cdot x + (-dx) \cdot y + dx \cdot c = 0
\]

\[
\therefore \quad Ax + By + C = 0 = F(x, y)
\]

where \( A = dy \quad B = -dx \quad C = dx \cdot c \)

Note: Essentially the implicit form of the line equation.
Idea is simple.

\[
\text{(Case I)} \quad (x_p, y_p)
\]

\[
\text{(Case II)} \quad \text{Say at step } n, \text{ we have marked}
\]

\[
\text{For the line to be continuous, there are only two choices for } (n+1)\text{th step.}
\]

(a) \(E \rightarrow (x_p + 1, y_p)\)

(b) \(NE \rightarrow (x_p + 1, y_p + 1)\)

Question is which one should we chose?

\[
M = (x_p + 1, \frac{y_p + 1}{2})
\]

\[
\text{Case I:} \quad \text{If the line passes below } M,
\]

\[
\text{choose } E.
\]

\[
\therefore M \text{ should be above line.}
\]

\[
\text{given by } F(M) < 0
\]

\[
i.e. \quad \text{If } F(x_p + 1, \frac{y_p + 1}{2}) < 0
\]

\[
\text{choose } E.
\]
Case II

Line passes above \( M \),

**Choose NE**

\[ P(M) \geq 0 \]

\[ \therefore \text{If } P(x_p+1, y_p+\frac{1}{2}) \geq 0 \]

**Choose NE.**

\[ \therefore d = P(x_p+1, y_p+\frac{1}{2}) \text{ is the decisive factor.} \]

**Note:** Decision made based on only comparisons.

**More optimizations**

To make the method efficient, we want to compute \( d \) in step \((n+1)\) incrementally from \( d \) computed in step \( n \).

\[ d_{(p,n+1)} = P(x_p+1, y_p+\frac{1}{2}) \]

\[ = A(x_p+1) + B(y_p+\frac{1}{2}) + C \]
Case I

If $E$ is chosen.

$$d_{\text{new}} = F(x_p+2, y_p+1/2)$$
$$= A - (x_p+2) + B - (y_p+1/2) + C$$
$$= A - (x_p+1) + A$$
$$+ B - (y_p+1/2) + C$$
$$= d_{\text{prev}} + A$$
$$= d_{\text{prev}} + dy$$

\[\therefore\text{ Update } d \text{ by adding only } dy.\]

\[\therefore\text{ Integer Operation.}\]

Case II

If $NE$ is chosen.

$$d_{\text{new}} = F(x_p+2, y_p+3/2)$$
$$= A - (x_p+2) + B - (y_p+3/2) + C$$
$$= A - (x_p+1) + B - (y_p+1/2) + C + A + B$$
$$= d_{\text{prev}} + A + B$$
$$= d_{\text{prev}} + dy - dx$$
Update \( d \) by \((dy-dx)\)

**Algorithm**

At each step

* If \( d < 0 \) choose \( E \)
  * else choose \( NE \).

Update \( d \).

**Initialize \( d \)**

Say the first pt is \((x_0,y_0)\)

\[
d = A(x_0+1) + B(y_0+1/2) + C = A x_0 + B y_0 + C + A + B/2
\]

Since \((x_0,y_0)\) is on the line,

\[
d = A + B/2 = dy - dx \frac{2}{2}
\]

Note that this is not integer.

But starting with \(2d\) is fine too.

Since we are checking the sign of \(d\), \(2d\) is not going to make a difference since sign remains same with scaling.

\[
: 2d = 2dy - dx \Rightarrow now a integer.
\]
Also note that you have to also scale the increments from \( d(p_{\text{prev}}) \) to \( d(\text{new}) \).

If \( E \) is chosen
\[
2 \, d(\text{new}) = 2 \, d(p_{\text{prev}}) + 2 \, dy
\]

If \( NE \) is chosen
\[
2 \, d(\text{new}) = 2 \, d(p_{\text{prev}}) + 2 \, dy - 2 \, dx
\]

**Example**

\[
(x_0, \, y_0) = (5, 8) \]
\[
(x_1, \, y_1) = (9, 11)
\]

\[
y = \frac{3}{4} \, x + \frac{17}{4}
\]

\[
r \, 3x - 4y + 17 = 0
\]

\[
A = dy = 3
\]
\[
B = -dx = -4
\]

\[
\therefore \, dx = 4
\]
1. \( d = 2dy - dx = 2 \times 3 - 4 = 2 > 0 \) 
   \( \Rightarrow \text{NE} \)

2. \( d = 2 + 2dy - 2dx = 2 + 6 - 8 = 0 \Rightarrow \text{NE} \)

3. \( d = 0 + 2dy - 2dx = 6 - 8 = -2 < 0 \Rightarrow E \)

4. \( d = 6 + 2dy = 6 + 6 = 12 \Rightarrow \text{NE} \)