Other Non-Linear Filters
Corner Detection (Non-linear filter)

- Corners have more edges than lines
- Should be easier
- But edge detectors fail – why?
  - Right at corner, gradient is ill-defined
  - Near corner, gradient has two different values
Moravec Operator

- Self-similarity
  - How similar are neighboring patches largely overlapping to me?
- Most regions - Very similar
- Edges - Not similar in one direction (perpendicular to edge)
- Corners – not similar in any direction
- Interest point detection – not only corners
Measuring self-similarity

- SSD = Sum of squared differences
- Corner is local maxima

\[ V = \sum_{i=1}^{9} (A_i - B_i)^2 = 2 \times 255^2 \]

\[ V = \sum_{i=1}^{9} (A_i - B_i)^2 = 3 \times 255^2 \]
Limitations

• Sensitive to noise
  – Responds for isolated pixel
• Larger patches for robustness
Limitations

- Responds also to diagonal edges
Limitations

- Anisotropic (Not rotationally invariant)
Harris & Stephens/Plessey Corner Detector

- Consider the differential of the corner score with respect to direction
- Describes the geometry of the image surface near the point \((u,v)\)

\[
A = \sum_u \sum_v w(u, v) \begin{bmatrix}
I_x^2 & I_x I_y \\
I_x I_y & I_y^2
\end{bmatrix} = \begin{bmatrix}
\langle I_x^2 \rangle & \langle I_x I_y \rangle \\
\langle I_x I_y \rangle & \langle I_y^2 \rangle
\end{bmatrix},
\]

Hessian Matrix
(Second derivatives of multi-variate function)
How to find the corner?

- The eigenvalues are proportional to the principal curvatures
- If both small, no edge/corner
- If one big and one small, edge
- If both big, then corner
Rotationally Invariant

- If $w$ is Gaussian, then this is isotropic

\[
A = \sum_u \sum_v w(u, v) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = \begin{bmatrix} \langle I_x^2 \rangle & \langle I_x I_y \rangle \\ \langle I_x I_y \rangle & \langle I_y^2 \rangle \end{bmatrix},
\]

![Image of a scene with objects and a pattern]
Median filter

- Replace by median of the neighborhood
- No new gray levels
- Removes the odd man out
  - Good for outlier removal
- Retains edges
Median filter

<table>
<thead>
<tr>
<th>INPUT</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>.......</td>
<td>..</td>
</tr>
<tr>
<td>.......</td>
<td>..</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MEDIAN</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>.......</td>
<td>..</td>
</tr>
<tr>
<td>.......</td>
<td>..</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MEAN</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>.......</td>
<td>..</td>
</tr>
<tr>
<td>.......</td>
<td>..</td>
</tr>
</tbody>
</table>
Salt and Pepper Noise
Difference from Gaussian Noise

Gaussian

Salt and Pepper
Median Filter

original image

1px median filter

3px median filter

10px median filter
Comparison

original

added noise

average

median