Analog Signals

- Both independent and dependent variables can assume a continuous range of values
- Exists in nature

Digital Signals

- Both independent and dependent variables are discretized
- Representation in computers
- **Sampling**
  - Discrete independent variable
  - Sample and hold (S/H)
- **Quantization**
  - Discrete dependent variable
  - Analog to Digital Converter (ADC)
Digital Signal

Sampled Signal
Digitized Signal

- Depends on number of bits
- 12 bits = 4095 levels
- $0.0 \leq \text{Voltage} \leq 4.096$
- 2.56 and 2.5601 TO 2560
- Each level (LSB) = 0.001
- Error $\leq \pm \frac{1}{2}$ LSB
- Called Quantization Error
Quantization Error

- Usually like random noise
- Noise is present in most signal acquisition systems
- Random uncorrelated samples added to the original signal

Proper Sampling

- If the original signal can be reconstructed *unambiguously* from the sampled signal

\[
\text{Cycles/Sample} = \frac{\text{Number of cycles per second}}{\text{Number of samples per second}} = \frac{\text{Analog Frequency}}{\text{Sampling Rate}}
\]
Is it Proper Sampling?

- DC signal
- Freq = 0.0 x Sampling Rate
- Proper

Is it Proper Sampling?

- Freq = 0.09 x Sampling Rate
- Each sample covers 0.09 cycles
- Proper
Is it Proper Sampling?

- Freq = 0.31 x Sampling Rate
- Larger fraction of cycles per sample
- Proper

![Graph showing proper sampling example](image)

Is it Proper Sampling?

- Freq = 0.95 x Sampling Rate
- Much larger parts of cycles per sample
- Not Proper
- Aliasing
- Changes frequency and phase

![Graph showing improper sampling example](image)
Sampling Theorem

- Proper Sampling: At least one sample per half cycle
- $\text{Freq} \leq 0.5 \times \text{Sampling Rate}$
- $\text{Sampling Rate} \geq 2 \times \text{Frequency}$
- Nyquist Rate

Time (Spatial) Domain vs. Frequency Domain

- Any one dimensional analog signal can be represented as a linear combination of sine waves of different frequencies
1D Signal

- Example: Once scan line of an image
- Amount of each sine wave defined by its amplitude and phase

Representation in Both Domains

Time Domain

Frequency Domain
Representation in Both Domains

Time Domain

Frequency Domain

Extending it to 2D

Amplitude

Phase
Amplitude

- **Amplitude**
  - **How** much details?
  - Sharper details signify higher frequencies
  - Will deal with this mostly

Phase

- **Where** are the details?
- Though we do not use it much, it is important, especially for perception
Low Pass Filtering

Hierarchical Image Filtering
Hierarchical Filtering

<table>
<thead>
<tr>
<th>1/4</th>
<th>1/4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/4</td>
<td>1/4</td>
</tr>
</tbody>
</table>

1 x 1

N/4 x N/4
N/2 x N/2
N x N

Gaussian Pyramid
Band-limited Images
(Laplacian Pyramid)

\[ f_{n-2} < f_{n-1} < f_n \]

\[ B_{n-1} = G_{n-1} - G_{n-2} \]

\[ B_n = G_n - G_{n-1} \]

\[ f_{n-2} \quad f_{n-1} \quad f_n \]

\[ G_{n-2} \]

\[ G_{n-1} \]

\[ G_n \]
Edge Crispening

Second Order Edge Detection

A. The image
B. Image after convolution
C. Segmented convolved image
D. Edge detected image
Scaling Problem

- Edges in coarser level do not disappear in finer levels
- New edges are added
- Coarser level edges are most important
- Advances like a hierarchy