CS111 - Spring 2012 - WA1 (key)

1. (a) Linear $\quad y[i] = \dfrac{x[i-1] + x[i] + x[i+1]}{3}$

$\boxed{4+3+3}$

(i) Scaling or Homogeneity: In case of kx

$$\text{output} = \frac{kx[i-1] + kx[i] + kx[i+1]}{3} = k \cdot y[i]$$

(ii) Superposition or Additivity: $x_1 + x_2$

$$y[i] = \frac{1}{3}\left(\sum_{k=i-1}^{i+1} x_1[k] + x_2[k] \right)$$

$$= \frac{1}{3} \sum_{k=i-1}^{i+1} x_1[k] + \frac{1}{3} \sum_{k=i-1}^{i+1} x_2[k]$$

$$= y_1[i] + y_2[i]$$

Shift invariance

(iii) $x[i+s]$; output $= \frac{1}{3}\left(x[i+s-1] + x[i+s] + x[i+s+1]\right) = y[i+s]$

(b) $\frac{1}{3}\left(\delta[n-1] + \delta[n] + \delta[n+1] \right)$

(c) $\frac{1}{5}\left(\delta[n-2] + \delta[n-1] + \delta[n] + \delta[n+1] + \delta[n+2] \right)$

2. (a) $x[n] * h[n]$

$\boxed{4\times 3}$

$$= \delta[n-a] * \delta[n-1] + \delta[n-a] * \delta[n+1] + \delta[n-b] * \delta[n-1] + \delta[n-b] * \delta[n+1]$$

$$= \delta[n-a-1] + \delta[n-a+1] + \delta[n-b-1] + \delta[n-b+1]$$

(b) $x[n] * h[n] = e^{n} * \delta[n+2] = e^{n+2}$

(c) $x[n] * h[n] = e^{-n} * \delta[n-2] = e^{-n+2}$

(d) $x[n] * h[n] = e^{-n} * \delta[n] - e^{-n} * \delta[n-1]$

$$= e^{-n} - e^{-n+1} = e^{-n}(1-e)$$

$\boxed{7}$

2+3

3. (a)

	$x-1$	x	$x+1$
y	-1	+1	0

(b)

y	0	-1	1

(c)

	$x-1$	x	$x+1$
$y-1$	-1	0	1
y	-2	0	+2
$y+1$	-1	0	1

2+3+3

2+3

4. (a) impulse response

$a[n] = \delta[n]$ ← unit impulse

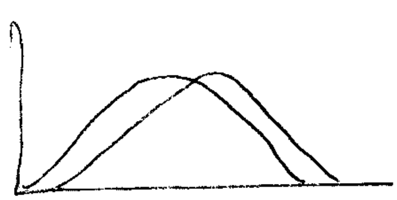
(b) inverted output $= -b[n]$

(c) $a[n] + b[n] = \delta[n] + b[n]$

(d) $a[n] - b[n] = \delta[n] - b[n]$

(e) (d) is a high pass filter

(f) For c, in case of low freq



ghosting artifact

for d,

5.

$\boxed{15}$

$4+3+4+4$

blurs $B = \begin{array}{|c|c|} \hline 1/4 & 1/4 \\ \hline 1/4 & 1/4 \\ \hline \end{array}$, horizontal gradient, $G = \begin{array}{|c|c|} \hline -1 & +1 \\ \hline -1 & +1 \\ \hline \end{array}$

(a) gradient is (in 1D)

$$g[n] = x[n-1] - x[n]$$

i) Homogeneity: $k\,x[n]$

gradient $= k\,x[n-1] - k\,x[n] = k\,g[n]$

ii) Additivity: $x[n] = x_1[n] + x_2[n]$

gradient $= x_1[n-1] + x_2[n-1] - x_1[n] - x_2[n]$

$= g_1[n] + g_2[n]$

iii) Shift invariance: $x[n+s]$

gradient $= x[n+s-1] - x[n+s] = g[n+s]$

(b) Convolution is commutative, so same output

(c) $S = B * G$

(d) $I' = I * B + I * G$

$= I * (B + G)$

$\therefore\ S = B + G$

(a) curvature based → laplacian ; so it will take single conv
6.(c) (b) others multiple convolution

The low pass filter is employed to remove high frequency noise from a digital image i.e it is employed for smoothing before applying the curvature or gradient filter.

6.(d)

After passing through the low-pass filter for smoothing, the pixel values of the resulting signal will no longer be independent. Rather, smobthing was used as a method to predict a pixel's value from the values of it's neighbors. However, if we increase the width of the low pass filter, then the pixels will tend to look more like their neighbors and the derivatives will become smaller (because they measure the tendency of pixels to look different from their neighbors).

~~6(a)~~

~~C will require multiple convolution operation. The output value of the edge finding detector in pixel (i,j) is~~

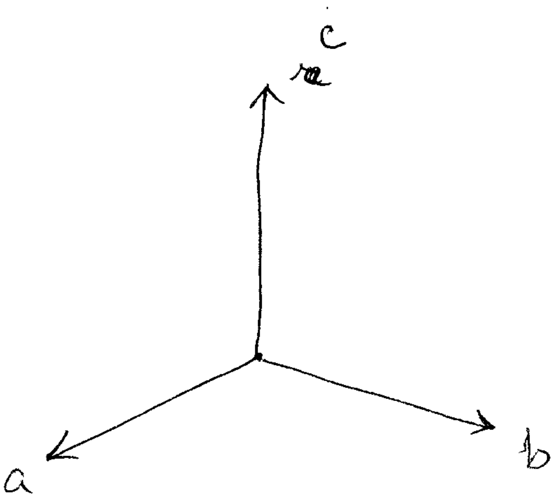
$$\cancel{f(i,j) = \max\left[0, \max_k (g * h_k)\right]}$$

~~For a particular size, we have to consider all possible orientations.~~

~~6(b)~~ 6 (a & b)

The gradient operators which are rotationally invariant (e.g. Laplacian) are computed from single mask only. Others, use multiple masks.

7. (a) 3D



(b) Initialize

For each edge pixel (x, y) in image
    for each radius r
        For $\theta = 0$ to 360

$$a = x + r\cos\theta$$
$$b = y + r\sin\theta$$
$$c = r$$

$$H(a,b,c) \mathrel{+}= 1$$

Or,
$$x = a + c\cos\theta$$
$$y = b + c\sin\theta$$
$$z = c$$

        end
    end
end

(c)