CS 111: Digital Image Processing (Fall 2016)
Written Assignment 2

1) Consider the box filter *in spatial domain* for a low pass filter. **[2+3+4+2+3+3+4=21]**

   a. What is its frequency domain response?
      Sinc
   b. Is the box filter an ideal low pass filter? Justify your answer.
      No. Ideal box filter won't allow any high frequencies. But a multiplication with sinc still allows leakage high frequencies.
   c. Is a box filter *in the frequency domain* an ideal low pass filter? Justify your answer.
      No. A box in frequency domain is a sinc in spatial domain which is an infinite function and cannot be represented as a digital kernel.
   d. What is the frequency domain response of a Gaussian filter *in the spatial domain*?
      Gaussian.
   e. How does it compare to box filter *in spatial domain* for low pass filtering? Justify your answer.
      It has much better cut off than box filter and hence is a better low pass filter.
   f. A multiplication of Gaussian and Sinc *in the spatial* domain is considered an ideal low pass filter. Express analytically the frequency domain response of this filter.
      The multiplication in spatial domain is a convolution in frequency domain. Therefore, Gaussian multiplied by Sinc in spatial domain is a Gaussian convolved with Box in frequency domain.
   g. How does this filter compare with the Gaussian filter in the spatial domain? Justify your answer. (Hint: Use pictures of the frequency domain response to identify pros and cons)
      Gaussian convolved with Box in frequency domain is like a smoothing of the Gaussian using a box filter. Therefore, this filter will have the cut off frequency higher than a Gaussian (cons), but its attenuation of the higher frequencies will be lower than that of the Gaussian (pros).

2) Supposed that you form a low-pass spatial filter *h(x,y)* that averages all the eight immediate neighbors of a pixel *(x,y)* but excludes itself. **[5+5=10]**

   a. Find the equivalent frequency domain filter *H(u,v)*.
      Note that this is equivalent to convolving with a box filter (which is the average of all the nine neighbors), scale it by 9/8, and then subtract 1/8 of the image from it.
      9/8(I*B) – 1/8(I*∂) = I * (9/8B – 1/8∂)
      This is in spatial domain. The frequency domain response will be 9/8Sinc – 1//8.
   b. Show that your result is again a low-pass filter.
      Draw the picture and it is still a low pass filter.

3) Any high pass filter has a strong spike at the origin. Explain the source of these spikes. **[10]**
   They have to derive the high pass from the low pass to show the source of the spike.

4) You have an image of bandwidth 100Hz. What is the minimum resolution of the display than can display this image free of artifacts? How will you process this image to make this suitable to display on a 50x50 resolution display? Justify your answers. **[5+5=10]**
   200 x 200
   Maximum frequency that can be sampled by 50 x 50 is 25 Hz. Therefore, we will have to low pass filter the image with cut off frequency at 25 Hz.

5) An image has a probability density function (PDF) of p(r) = 2(1-r). We want to transform this image so that its PDF becomes p(z) = 2z. Assume continuous images and find the transformation (in terms of r and z) that would achieve this goal. **[10]**
   X3 = ∫s3xdl = ∫s1s2xdl

X1 = ∫s1xdl
X2 = ∫s2xdl
X1X2 = ∫s1s2x^2dl ≠X3

6) You want to digitize an analog signal of bandwidth 120Hz. The sampling frequency of your display is 100 Hz. The bandwidth of your reconstruction kernel is 80 Hz. **[5x4=20]**

   c. Why won't you be able to sample and reconstruct this signal without artifacts using this display?
   Since the bandwidth and the sampling frequency smaller than the bandwidth, the copies formed in the frequency domain during the sampling process would overlap creating aliasing artifact.
   d. How would you process the image to reconstruct it without any artifacts?
   Low pass filter the image at 100Hz.
   e. What kind of artifacts would the reconstruction kernel generate?
   The bandwidth of the kernel is smaller. Therefore, some high frequencies will be dropped giving a blurring artifact.
   f. How would you change the reconstruction kernel to correct it?
   The reconstruction kernel should also match with the bandwidth to be 100Hz.

7) When we mix blue paint with yellow paint we get green. But when we project blue light on yellow light, we get brown. How do you explain this contradiction? **[5]**
   This is due to additive and subtractive color mixture. Combining two colors in subtractive color mixtures means the part of the spectrum not reflected by both is due to being absorbed by either one. Therefore, yellow absorbs blue part of the spectrum while blue absorbs red part of the spectrum. What is reflected by both is green part. But when yellow and blue lights are added, the combination is all wavelengths in these regions leading to brown.

8) Consider a linear display whose red, green and blue primaries have chromaticity coordinates of *(0.5, 0.4)*, *(0.2, 0.5)* and *(0.1, 0.1)* respectively. The maximum intensity (defined by X+Y+Z) of white is $1000 cd/m^2$ respectively. The white point of the display is (0.33,0.37) Generate the matrix that converts the RGB coordinates for this device to the XYZ coordinates. What is the XYZ coordinates of the color generated by the RGB input (0.5, 0.75, 0.2) on this device? **[10]**
   A(0.5,0.4) + B(0.2, 0.5) + (1-A-B)(0.1, 0.1) = (0.33, 0.37)
   Gives two equations
   40A+10B=23
   30A+40B=27
   Or
   160A+40B=92
   30A+40B=27
   Subtracting we get
   130A = 65
   Or A=1/2=0.5
   Therefore, B = 0.3 and (1-A-B) = 0.2, i.e. the proportions of the intensity of R, G and B in the intensity of white is 0.5, 0.3 and 0.2 respectively. Therefore, intensity of R, G and B are given by 500, 300 and 200 respectively.
   Now, We know red chromaticity is (0.5,0.4) and intensity is 500. Therefore its tristimulus value is given by (250, 200, 50).
   Similarly, we get the tristimulus values of green as (60,150, 90) and that of blue as (20, 20, 160).

9) The spectrum of color $C_1 = (X_1,Y_1,Z_1)$ and $C_2 = (X_2,Y_2,Z_2)$ are given by $s_1(\lambda)$ and $s_2(\lambda)$ respectively. Let the color formed by multiplications of the spectrums s_1 and s_2 be s_3, i.e. $s_3(\lambda) = s_1(\lambda) * s_2(\lambda)$. Is it true that the XYZ coordinate corresponding to s_3, denoted by C_3, is (X_1X_2,Y_1Y_2,Z_1Z_2)? Justify your answer with calculations. **[5]**

$X3 = \int s3xdl = \int s1s2xdl$
$X1 = \int s1xdl$
$X2 = \int s2xdl$
$X1X2 = \int s1s2x^2dl \neq X3$

10) An image has a probability density function (PDF) of p(r) = 2(1-r). We want to transform this image so that its PDF becomes p(z) = 2z. Assume continuous images and find the transformation (in terms of r and z) that would achieve this goal. **[10]**
$\int$2(1-r)dr = 2r – r^2/2
$\int$2zdz = z^2
Thereforem z = sqrt(2r-r^2/2)

11) C_1 and C_2 are colors with chromaticity coordinates *(0.33, 0.12)* and *(0.66, 0.66)* respectively. In what proportions should these colors be mixed to generate a color C_3 of chromaticity coordinates *(0.55, 0.48)*? If the brightness of C_3 is *90*, what are the brightness of C_1 and C_2? **[10]**

This is like question 2.
A(0.33,0.12) + (1-A)(0.66, 0.66) = 0.55
From this we get
33A + 66 – 66A = 55
Or, 33A = 11,
Or, A = 1/3
Therefore (1-A) = 2/3
This means that the proportions of the intensity of C1 and C2 is 1/3 and 2/3 respectively.
The brightness of C3 is 90. Therefore the proportions give the brightness of C1 and C2 as 30 and 60 respectively.

However, chromaticity coordinates cannot really be (0.66,0.66), but the math should work out anyway.