1. An audio signal makes 250 cycles in its span (or has a frequency of 250Hz). How many samples do you need, at a minimum, to sample it correctly? [1]

500 samples (since Nyquist sampling needs to be at least twice the frequency)

2. If the number of bits is reduced, the quantization error increased [1]

Step size will be increased and hence quantization error is also increased.

3. Draw a 3x3 kernel that will take an image as input and output as image which has a quarter dimmed ghost to its bottom right. Make sure that the total energy is preserved. [3]

In Matlab notation, the kernel is \([4/5, 0, 0; 0, 0, 1/5]\). The goal is to create the image plus itself at quarter brightness at the bottom right. Since we have to preserve energy, the sum of the fractions should add up to 1. Therefore, \(X + 1/4X = 1\)
From this we get, \(x = 4/5\). And hence the kernel. Also, note that if we do not have no contribution or 0 value between the 4/5 and the 1/5 in the kernel, then it will become just another low pass filter where the contribution is spread to only one neighboring pixel (instead of two or four that we have done in class). Therefore, we need to have the second row to be completely 0. This will assure ghosting and not blurring.

4. Let us consider a signal \(x(t)\). Consider the operation \(\frac{1}{2}(x(t) \ast \partial(t) + x(t) \ast \partial(t-1))\). This is equivalent to [3]
   - low pass filtering of \(x\)
   - high pass filtering of \(x\)
   - band pass filtering of \(x\)

5. Calculate the convolution of the following signals. [2]

   \[h[t] = \partial[t+1], \quad x[t] = \partial[t-a] + \partial[t+b]\]

   Based on delay property of convolution we have
   \[x[t] \ast \partial[t+s] = x[t+s]\]

   So we have
   \[(\partial[t-a] + \partial[t+b]) \ast \partial[t+1] = \partial[t-a+1] + \partial[t+b+1]\]

6. [1] Laplacian pyramid provides
   - low pass filtering
   - high pass filtering
   - band pass filtering
7. [2] Consider a checkerboard (black and white squares like a chess board) image rotated 45 anti-clockwise degrees. In which direction would you find the maximum values (whites) in the frequency plot? [Mark all that apply]
   - Horizontal
   - Vertical
   - 45 degrees
   - 135 degrees

The values will be in a perpendicular direction to edges. Horizontal edges become 135 degree edges after rotation and vertical ones become 45 degrees. Therefore, the two directions perpendicular to this is 45 and 135 degrees.

8. [2] Why are we usually only concerned about the magnitude plot in frequency domain and not the phase?
   Phase information is better studied in the spatial domain since the synchronicity of phase of one wave with respect to other is better captured in spatial domain.

9. [1] The frequency domain response of a comb function is a
   - Comb function
   - Delta
   - Gaussian

10. [2] A box filter in spatial domain is not the ideal filter due to
    - Infinite support
    - Leakage frequency
    - Aliasing artifacts

11. [2] The low pass filter is a linear operation. Given this prove that the high pass filter is also a linear operation.
    As it is shown in slides we can achieve high pass filter by subtracting the low passed image from the original image. So we have
    \[ I_{\text{original}} - I_{\text{low passed}} = I - I \ast G = I \ast \partial - I \ast G = I \ast (\partial - G) \]

12. [3] Match following images with corresponding Fourier transform
13. [3] The Harris corner detector is invariant to which of the following transformations
   i. Scaling
   ii. Translation
   iii. Rotation

   i. First derivative is maximum
   ii. Second derivative is maximum
   iii. First derivative is zero
   iv. Second derivative is zero
15. [2] What are the four steps of Canny edge detector?
   - Suppress noise using low pass filtering
   - Computer gradient magnitude and direction image
   - Apply non-maxima suppression
   - Use hysteresis and connectivity analysis

16. [2] Consider a 2D square whose center is at (2,2). The transformation to be applied to achieve a scaling of the square by 2 in the X direction and 3 in the Y direction is given by
   - $T(2,2)S(3,2)T(-2,-2)$
   - $T(-2,-2)S(3,2)T(2,2)$
   - $T(2,2)S(2,3)T(-2,-2)$
   - $T(-2,-2)S(3,2)T(2,2)$

17. [1] The intrinsic parameter matrix of a camera is a
   - Lower Triangular matrix
   - Upper Triangular matrix
   - Symmetric matrix

18. [1] Consider a vector with direction $(x, y)$. Its homogeneous coordinates is given by
   - $(x, y, 1)$
   - $(x, y, 0)$
   - $(kx, ky, 0)$
   - $(kx, ky, 1)$


   Non-Linear Transformation ➔ Do not change the length and angles
   Affine Transformation ➔ Do not change the degree of the curves
   Euclidian Transformation ➔ Do not change the ratio of lengths and angles
   Projective Transformation ➔ Can change the degree of curves and surfaces
   Linear Transformation ➔ Can change parallel lines to intersecting lines

20. [1] A non-uniform scaling is applied to a sphere. The resulting 3D shape will be
   - Sphere
   - Ellipsoid
   - Paraboloid
   - Cube