Fundamentals

Data
Outline

• Visualization

• Discretization
  • Sampling
  • Quantization

• Representation
  • Continuous
  • Discrete

• Noise
Data

• Data: Function dependent on one or more variables.

• Example
  • Audio (1D) - depends on time $t$ - $A(t)$
  
  • Image (2D) - depends on spatial coordinates $x$ and $y$ - $I(x, y)$
  
  • Video (3D) - depends on spatial coordinate $(x, y)$ and time $t$ - $V(x, y, t)$
Visualization

- Plot of dependent variable with respect to independent ones
- 2D plot is a *height field*
Visualization

- Other kinds of visualizations
- Color image: three color channels: $R(x, y), G(x, y)$ and $B(x, y)$
Discretization

- Data exists in nature as a continuous function.
- Convert to discrete function for digital representation
  - Discretization
- Two concepts
  - Sampling
  - Quantization
Sampling

- Set of values of continuous $f(t)$ at specific values of $t$.
- Reduces continuous function $f(t)$ to discrete form
Uniform vs Non-uniform sampling
Reconstruction

- Get the continuous function from the discrete function

Sampling  Correct Reconstruction
Reconstruction

- Accurate reconstruction needs adequate samples
Aliasing

- Incorrect representation of some entity

A much lower frequency

Zero frequency
Nyquist Sampling Rate

- By sampling \textit{at least} twice the frequency (2 samples per cycle), signal can be reconstructed correctly.
Quantization

- A analog signal can have any value of infinite precision
- Digital signal can only have a limited set of value
An Alternative Representation

- **frequency domain representation**
  - A signal is a linear combination of sine or cosine waves

\[
\sum_{j=0}^{\infty} a_j \sin(j \omega_0 t) + b_j \cos(j \omega_0 t)
\]

- Signal or cosin or cosir

\[
\begin{align*}
\sin(2\pi ft) & \quad + \frac{1}{3}\sin(6\pi ft) \\
& + \frac{1}{5}\sin(10\pi ft) \\
& + \frac{1}{7}\sin(14\pi ft)
\end{align*}
\]

=
Representation

- **Explicit Representation**
  \[ y = mx + c \]

- **Implicit Representation**
  \[ ax + by + c = 0 \]

- **Parametric Equation**
  - Using one or more parameters
  - **Example:** point \( p \) on a line segment between two points \( P \) and \( Q \)
    \[ p = P + t(Q - P), \quad 0 \leq t \leq 1 \]
Discrete Representation

• A 3D cube defined by a set of quadrilaterals or triangles
  • This is called *Mesh*

• The entities that make up the mesh (e.g. lines, triangles or quadrilaterals) are called the *primitives*.

Vertices

\[
V_1 (x_1, y_1, z_1) \\
V_2 (x_2, y_2, z_2) \\
\vdots \\
V_8 (x_8, y_8, z_8)
\]

Triangles

\[
V_1, V_2, V_4; V_1, V_4, V_3; V_3, V_4, V_7; \\
V_4, V_8, V_7; V_4, V_2, V_8; V_2, V_6, V_8; \\
V_1, V_5, V_2; V_2, V_5, V_6; V_3, V_5, V_1; \\
V_3, V_7, V_5; V_5, V_7, V_6; V_6, V_7, V_8
\]
Properties

• **Geometric Properties:**
  • Position
  • Normal
  • Curvature

• **Topological Properties:** remains invariant to changes in geometric properties
  • Connectivity or Adjacency
  • Dimension
  • Manifold / non-manifold
  • Euler characteristic/Genus
Manifold Definitions

• **Manifold**
  • Every edge has exactly two incident triangles.

• **Manifolds with boundaries**
  • Every edge has either one or two incident triangles.

• **Non-manifold**
  • Not with above restrictions.
Euler Characteristic

- \( e = V - E + F \) (V: Vertices, E: Edges, F: Faces).
- Cube has 8 vertices, 12 edges, 6 faces
  - \( e = V - E + F = 8 - 12 + 6 = 2 \)
- Changing geometric properties keeps Euler characteristic invariant
  - Such as adding edges, vertices
Genus

- (Naïve) Number of “handles”.
- Relationship between e and g: $e = 2 - 2g$
  - Sphere, cube $g = 0$
  - torus, coffee cup $g = 1$
- Going from coffee cup to torus
  - Changing only geometric properties
Noise

- Addition of random values at random locations in the data.
  - Random noise
Noise

• *Outlier Noise*

• An example of such noise is *salt and pepper* noise

• Can be solved by Median filter
Noise

- Some noise look random in spatial domain but can be isolated to a few frequencies in spectral domain.
- Can be removed by Notch filter.
Techniques

• Interpolation
  • Linear Interpolation
  • Bilinear Interpolation
• Geometric Intersections
Interpolation

• Estimate function for values which it has not been measured.

• **Linear interpolation:**
  • Assuming a line between samples.
  • Change abruptly at sample points
  • $C^0$ continuity
Interpolation

- **Non-linear interpolation:**
  - A smooth curve passes through samples
  - First derivative continuous - $C^1$ continuous
  - Second derivative continuous - $C^2$ continuous
Linear Interpolation

• Point V on the line segment $V_1V_2$ is given by

$$V = \alpha V_1 + (1 - \alpha)V_2, \quad 0 \leq \alpha \leq 1$$

• Example: Linear interpolation of color at point $V$

$$C(V) = C(\alpha V_1 + (1 - \alpha)V_2) = \alpha C(V_1) + (1 - \alpha)C(V_2)$$
Bilinear Interpolation

- 2D Data

- Interpolating in one direction followed by interpolating in the second direction.

\[ F(Q) = (1 - \alpha)C + \alpha D \]

\[ F(R) = (1 - \alpha)A + \alpha B \]

\[ F(P) = F(Q)\beta + F(R)(1 - \beta) \]