Linear Filters
Blurring filters

• More blurring implies widening the base and shortening the height of the spike further.

• What does it look like?

• Box filters are not best blurring filters but the easiest to implement.
Frequency domain representation

- *frequency domain representation*
  - A signal is a linear combination of sine or cosine waves
    \[
    c(t) = \sum_{i=1}^{\infty} a_i \cos(f_i + p_i)
    \]
  - Signal can be represented by the coefficients of these sine or cosine waves
Time/Frequency or Primal/Dual

- Lower energy in higher frequencies
- Amplitude is more important
- Phase information is better studied in time domain
What happens in 2D?

Frequency = $(\sqrt{g} + h)^{1/2}$
Orientation = $\tan^{-1}(h/g)$
Duality

Spatial Domain

Frequency Domain
Duality

Spatial Domain

Frequency Domain

Widening in one domain is narrowing in another and vice-versa.
Duality

- Convolution of two functions in time/spatial domain is a multiplication in frequency domain

- Vice Versa
All Pass Filter

\[ x[t] * \delta[t] = x[t] \]
Low Pass Filter

$k[t]$

$K[f]$

$F$

$a(t)$

$A(f)$

$X$
Low Pass Filtering

- Box filter is known as *low pass filter*.
Box Filter

- Effect of increasing the size of the box filter
Gaussian Pyramid
Gaussian Pyramid

Level 0: $2^n \times 2^n$

Level 1: $2^{n-1} \times 2^{n-1}$

Level $n$: $1 \times 1$
Box is not the only shape

- Gaussian is a better shape
- Any thing more smooth is better
Hierarchical Filtering
Issue of Sampling

- As an image undergoes low pass filtering, its frequency content decreases.
- Minimum number of samples required to adequately sample the low pass filtered image is less.
- Low pass filtered image can be at a smaller size than the original image.
Subsampling

simple subsampling

pre-filtering and subsampling
Aliasing Artifact

- Input (256 x 256)
- Subsampled (128 x 128)
- Insufficient sampling. Hence, aliasing.
- Subsampled from filtered image (128 x 128)
- Filtered (256 x 256)
- Filtering reduces frequency content. Hence, lower sampling is sufficient.

ANTI-ALIASING
High Pass Filter

- subtract the low pass filtered image from the original image

\[ I_h = I - I \ast l \]
\[ = I \ast \partial - I \ast l = I \ast (\partial - l) \]
High Pass Filter

Original Image  Low pass filtered  High pass filtered
Band-limited Images (Laplacian Pyramid)

\[ f_{n-2} < f_{n-1} < f_n \]

\[ B_{n-1} = G_{n-1} - G_{n-2} \]

\[ B_n = G_n - G_{n-1} \]
Band-limited Images (Laplacian Pyramid)
2D Filter Separability

- Visualizing 2D filters from their 1D counterpart

Box Filter  Gaussian Filter  High Pass Filter
2D Filter Separability

- **Separability**
  - 2D filter \( h (p \times q) \) is separable if \( h \) can be separated into two 1D filters \( a \) and \( b \) such that

\[
h[i][j] = a[i] \times b[j]
\]

- Convolving image with \( h \) is same as convolving its rows with \( a \) and then its columns with \( b \)
2D Filter Separability

- **Advantage**
  - Separable filters can be implemented more efficiently

- Convolving with $h$
  - \( \text{Number of multiplications} = 2pqN \)

- Convolving with $a$ and $b$
  - \( \text{Number of multiplications} = 2(p+q)N \)