Fourier Transform in Graphics and Vision

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ICS 288

Frequency Based Representation of a Spatial Signal

- Any one dimensional function can be represented as a linear combination of sine waves of different frequencies
1D Signal

- Example: Once scan line of an image
- Amount of each wave defined by its amplitude and phase

Sine Waves form a Basis

- Forms a basis for the set of functions
  - Sine wave of one frequency is linearly independent of another
  - Any function is a linear combination of this linearly independent basis function
- Infinite Basis
  - When we do it in digital domain, we can find a finite basis
  - Matlab
Fourier Transform (FT)

SPATIAL DOMAIN

\[ h(x) \]

FREQUENCY DOMAIN

\[ A(f) \]

\[ P(f) \]

FT

IFT

Extending it to 2D

- Any 2D function (an image) can be represented as a linear combination of sine waves of different frequencies and orientation
Extending it to 2D

Amplitude

- **Amplitude**
  - **How** much details?
  - Sharper details signify higher frequencies
  - Will deal with this mostly
Phase

- **Where** are the details?
- Though we do not use it much, it is important, especially for perception.

Properties of Fourier Transform

- **Symmetric**

\[
\begin{align*}
    h(x) & \rightarrow H(f) \\
    H(x) & \rightarrow h(f)
\end{align*}
\]
Properties of Fourier Transform

SPATIAL DOMAIN

FREQUENCY DOMAIN

Convolution

Convolution Kernel

Imaging and Adding
Convolution and Multiplication

SPATIAL DOMAIN

\[ h_1(x) \quad h_2(x) \]

\[ x \quad \ast \quad x \]

FREQUENCY DOMAIN

\[ A_1(f) \quad A_2(f) \]

\[ f \quad \rightarrow \quad x \]

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Convolution and Multiplication

SPATIAL DOMAIN

\[ h_1(x) \quad h_2(x) \]

\[ x \quad \ast \quad x \]

FREQUENCY DOMAIN

\[ A_1(f) \quad A_2(f) \]

\[ f \quad \rightarrow \quad x \]

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**Convolution and Multiplication**

**SPATIAL DOMAIN**

\[ h_1(x) \ast h_2(x) = f \]

**FREQUENCY DOMAIN**

\[ A_1(f) \ast A_2(f) = \]

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**Image Filtering**

**SPATIAL DOMAIN**

\[ h_1(x) \ast h_2(x) \]

**FREQUENCY DOMAIN**

\[ A_1(f) \ast A_2(f) \]

Reduced the Bandwidth

= Low pass filter
Ideal Low Pass Filter

Types of Filters
Choosing Right Filters

- Should not pass high frequency
- Should be limited in spatial domain
- Box Filter in spatial domain
  - Not good since passes high frequencies
- Synch Filter in spatial domain
  - Infinitely long in spatial domain
- Gaussian is a good compromise

Hierarchical Image Filtering

\[
\begin{array}{c}
\text{h}_1(x) \quad \ast \quad \text{A}_1(f) \\
\text{h}_2(x) \quad \ast \quad \text{A}_2(f)
\end{array}
\]

SPATIAL DOMAIN

FREQUENCY DOMAIN
Hierarchical Filtering

1/4 1/4
1/4 1/4

N x N
N/2 x N/2
N/4 x N/4

Gaussian Pyramid

Image Pyramid
Low resolution
High resolution

Image Pyramid
Low resolution
High resolution

Image Pyramid
Frequency Domain
Low resolution
High resolution
Band-limited Images (Laplacian Pyramid)

\[ f_{n-2} < f_{n-1} < f_n \]

\[ B_{n-1} = G_{n-1} - G_{n-2} \]

\[ B_n = G_n - G_{n-1} \]

\[ G_{n-2} \]

\[ G_{n-1} \]

\[ G_n \]
Sampling and Reconstruction

Sampling

Convolution

-\( f_s \) \( f_s \) \( f_s \) \( f_s \) \( 2f_s \) \( 3f_s \)

Original signal
Sampling
Sampled signal
Reconstruction
Reconstructed signal
Reconstruction

Multiplication

Sampling

Convolution
Reconstruction

- $f_s$
- $f_s$
- $2f_s$
- $3f_s$

Multiplication

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Aliasing

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Aliasing

Signal is band-limited first
Reconstruction

Multiplication

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Reconstruction

Multiplication

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Blurring

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Blurring

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