Logic

- Mathematicians prove theorems.
- Can we make machine do it?

Problem # 11

- If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

- Y = mYthical R = moRtal
  M = maMmal H = Horned G = maGical

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Inference
= weapons in arsenal
+ how to use them

Important rules

- $\alpha \Leftrightarrow \beta$ iff $\alpha \Rightarrow \beta$, $\beta \Rightarrow \alpha$
- $\alpha \Rightarrow \beta$ iff $\neg \alpha \vee \beta$
- $\neg(\alpha \wedge \beta) = \neg \alpha \vee \neg \beta$
- $\neg(\alpha \vee \beta) = \neg \alpha \wedge \neg \beta$
- $(\alpha \wedge \beta) \vee \gamma = (\alpha \vee \gamma) \wedge (\beta \vee \gamma)$
- $(\alpha \vee \beta) \wedge \gamma = (\alpha \wedge \gamma) \vee (\beta \wedge \gamma)$

Can be proved by truth table.

Important rules

- And many more.
- The more you know, the better you can do inference (Refer to textbook.)

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$$Y \Rightarrow \neg R, \qquad \neg Y \Rightarrow (R \wedge M),$$
$$(\neg R \vee M) \Rightarrow H, \qquad H \Rightarrow G$$

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- Y = mYthical R = moRtal
  M = maMmal H = Horned G = maGical

- Given these sentences, can we prove "if the unicorn is mortal, it is magical" , i.e. $R \Rightarrow G$?

$$Y \Rightarrow \neg R, \qquad \neg Y \Rightarrow (R \wedge M),$$
$$(\neg R \vee M) \Rightarrow H, \qquad H \Rightarrow G$$

Is $R \Rightarrow G$ true?

$$Y \Rightarrow \neg R, \qquad \neg Y \Rightarrow (R \wedge M),$$
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Knowledge Base

- $KB \models (R \Rightarrow G)$?

Is $R \Rightarrow G$ true?

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Knowledge Base

- $KB \models (R \Rightarrow G)$?
- Yes.

Is $R \Rightarrow G$ true?

$$Y \Rightarrow \neg R, \qquad \neg Y \Rightarrow (R \wedge M),$$
$$(\neg R \vee M) \Rightarrow H, \qquad H \Rightarrow G$$

Knowledge Base

- $KB \models (R \Rightarrow G)$?
- Yes.
- $R \Rightarrow \neg Y \Rightarrow M \Rightarrow H \Rightarrow G$

$\neg Y \Rightarrow R$

Soundness

- $R \Rightarrow \neg Y \Rightarrow M \Rightarrow H \Rightarrow G$
- Our inference is sound, since it's based on sound rules. (Our weapons are invincible!)

Completeness

- Inference is complete if it can be used to derive all possible (true) sentences.
- This is determined by truth table.
- Sound & Complete inference is hard
  - resolution

Proof by contradiction

- $KB \models \alpha$ iff

  $KB \Rightarrow \alpha$ iff hard to prove.

  $\neg KB \vee \alpha$

- Let's prove
  $\neg(\neg KB \vee \alpha)$ i.e. $KB \wedge \neg \alpha$ is false.

easy to prove using resolution

Digress:Problem #10 (a)

From Mid-term Winter 2012

- $(A \vee B \vee \neg C \vee D) \wedge (A \vee \neg D \vee E \vee F)$

Problem #10 (a)

From Mid-term Winter 2012

- $(A \lor B \lor \neg C \lor D) \land (A \lor \neg D \lor E \lor F)$

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- $(A \vee B \vee \neg C \vee D) \wedge (A \vee \neg D \vee E \vee F)$

  $\Leftrightarrow (A \vee B \vee \neg C \vee A \vee E \vee F)$

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Problem #10 (a)

From Mid-term Winter 2012

- $(A \vee B \vee \neg C \vee D) \wedge (A \vee \neg D \vee E \vee F)$

$$\Leftrightarrow (A \vee B \vee \neg C \vee A \vee E \vee F)$$

$$\Leftrightarrow (A \vee B \vee \neg C \vee E \vee F)$$

Problem #10 (c)

From Mid-term Winter 2012

- $(\neg C) \wedge (C) \Leftrightarrow F$

Problem #10 (d)

From Mid-term Winter 2012

- $(A \lor B \lor \neg C \lor D) \land (A \lor C \lor \neg D \lor E \lor F)$

Problem #10 (d)

From Mid-term Winter 2012

- $(A \lor B \lor \neg C \lor D) \land (A \lor C \lor \neg D \lor E \lor F)$

Problem #10 (d)

From Mid-term Winter 2012

- $(A \vee B \vee \neg C \vee D) \wedge (A \vee C \vee \neg D \vee E \vee F)$

  $\Leftrightarrow (A \vee B \vee D \vee \neg D \vee E \vee F)$

Problem #10 (d)

From Mid-term Winter 2012

- $(A \lor B \lor \neg C \lor D) \land (A \lor C \lor \neg D \lor E \lor F)$

  $\Leftrightarrow (A \lor B \lor D \lor \neg D \lor E \lor F)$

Problem #10 (d)

From Mid-term Winter 2012

- $(A \lor B \lor \neg C \lor D) \land (A \lor C \lor \neg D \lor E \lor F)$

  $\Leftrightarrow (A \lor B \lor D \lor \neg D \lor E \lor F)$

  $\Leftrightarrow (A \lor B \lor T \lor E \lor F)$

Problem #10 (d)

From Mid-term Winter 2012

- $(A \vee B \vee \neg C \vee D) \wedge (A \vee C \vee \neg D \vee E \vee F)$

  $\Leftrightarrow (A \vee B \vee D \vee \neg D \vee E \vee F)$

  $\Leftrightarrow (A \vee B \vee T \vee E \vee F)$

  $\Leftrightarrow T$

Resolution

- Resolution requires CNF
- KB $\wedge \neg \alpha$ CNF, if KB is CNF
- Turn KB into CNF!

- Given these sentences, can we prove "unicorn is both magical and horned" , i.e. $G \wedge H$?

$$Y \Rightarrow \neg R, \qquad \neg Y \Rightarrow (R \wedge M),$$
$$(\neg R \vee M) \Rightarrow H, \qquad H \Rightarrow G$$

$$Y \Rightarrow \neg R, \qquad \neg Y \Rightarrow (R \wedge M),$$
$$(\neg R \vee M) \Rightarrow H, \qquad H \Rightarrow G$$

- turn into CNF
- $Y \Rightarrow \neg R$ iff $(\neg Y \vee \neg R)$
- $\neg Y \Rightarrow (R \wedge M)$ iff $Y \vee (R \wedge M)$ iff $(Y \vee R) \wedge (Y \vee M)$
- $(\neg R \vee M) \Rightarrow H$ iff $(R \wedge \neg M) \vee H$ iff $(R \vee H) \wedge (\neg M \vee H)$
- $H \Rightarrow G$ iff $(\neg H \vee G)$

$\neg Y \vee \neg R$	$Y \vee R$	$Y \vee M$	$R \vee H$	$\neg M \vee H$	$\neg H \vee G$	$\neg G \vee \neg H$

KB

$\neg \alpha$

$\neg Y \lor \neg R$ $Y \lor R$ $Y \lor M$ $R \lor H$ $\neg M \lor H$ $\neg H \lor G$ $\neg G \lor \neg H$

$\neg H$

$\neg Y \vee \neg R$ $\quad Y \vee R$ $\quad Y \vee M$ $\quad R \vee H$ $\quad \neg M \vee H$ $\quad \neg H \vee G$ $\quad \neg G \vee \neg H$

$\neg H$ $\quad \neg M$

$\neg Y \vee \neg R$ $\quad Y \vee R$ $\quad Y \vee M$ $\quad R \vee H$ $\quad \neg M \vee H$ $\quad \neg H \vee G$ $\quad \neg G \vee \neg H$

$\neg H$ $\quad \neg M$ $\quad R$

$\neg Y \vee \neg R$ $Y \vee R$ $Y \vee M$ $R \vee H$ $\neg M \vee H$ $\neg H \vee G$ $\neg G \vee \neg H$

$\neg H$ $\neg M$ R Y

$\neg Y \vee \neg R$ $\quad Y \vee R$ $\quad Y \vee M$ $\quad R \vee H$ $\quad \neg M \vee H$ $\quad \neg H \vee G$ $\quad \neg G \vee \neg H$

$\neg H$ $\quad \neg M$ $\quad R$ $\quad Y$ $\quad \neg R$

$\neg Y \vee \neg R$ $Y \vee R$ $Y \vee M$ $R \vee H$ $\neg M \vee H$ $\neg H \vee G$ $\neg G \vee \neg H$

$\neg H$ $\neg M$ (R) Y ($\neg R$) ()

"empty" means false, and this makes the entire proposition false, since all clauses are linked by conjunction.

$\neg Y \lor \neg R$ $Y \lor R$ $Y \lor M$ $R \lor H$ $\neg M \lor H$ $\neg H \lor G$ $\neg G \lor \neg H$

$\neg H$ $\neg M$ R Y $\neg R$ $()$

We can prove/ disprove "any" α in this way, hence this inference is complete.

"empty" means false, and this makes the entire proposition false, since all clauses are linked by conjunction.