

# Local Search Algorithms

This lecture topic Chapter 4.1-4.2

Next lecture topic  
Chapter 5

(Please read lecture topic material before and  
after each lecture on that topic)

# Outline

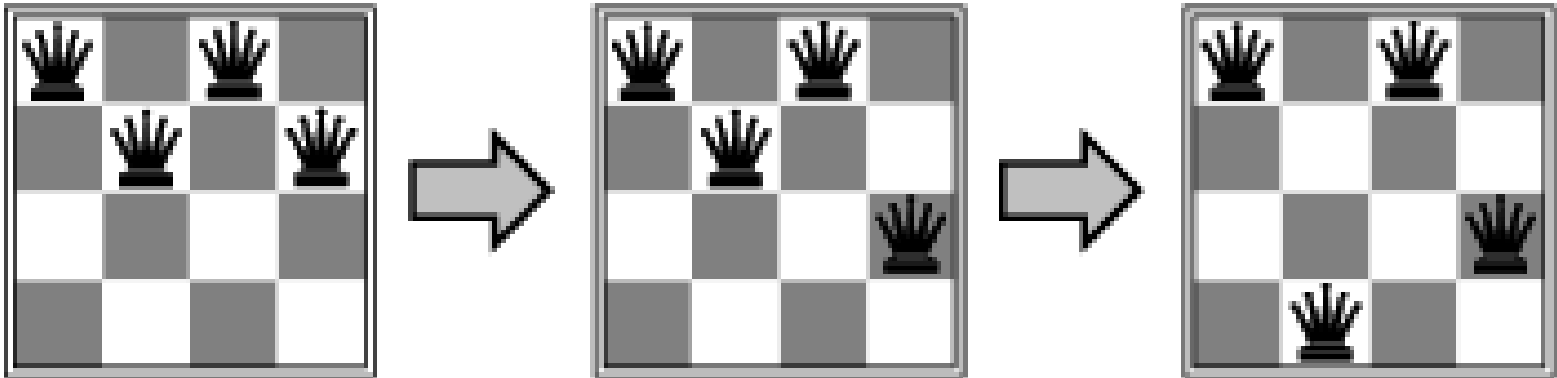
- Hill-climbing search
  - Gradient Descent in continuous spaces
- Simulated annealing search
- Tabu search
- Local beam search
- Genetic algorithms
- Linear Programming

# Local search algorithms

- In many optimization problems, the **path** to the goal is irrelevant; the goal state itself is the solution
- State space = set of "complete" configurations
- Find configuration satisfying constraints, e.g., n-queens
- In such cases, we can use **local search algorithms**
- keep a single "current" state, try to improve it.
- Very memory efficient (only remember current state)

# Example: $n$ -queens

- Put  $n$  queens on an  $n \times n$  board with no two queens on the same row, column, or diagonal



Note that a state cannot be an incomplete configuration with  $m < n$  queens

# Hill-climbing search

- "Like climbing Everest in thick fog with amnesia"

- ```
function HILL-CLIMBING(problem) returns a state that is a local maximum
  inputs: problem, a problem
  local variables: current, a node
                  neighbor, a node

  current ← MAKE-NODE(INITIAL-STATE[problem])
  loop do
    neighbor ← a highest-valued successor of current
    if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
    current ← neighbor
```

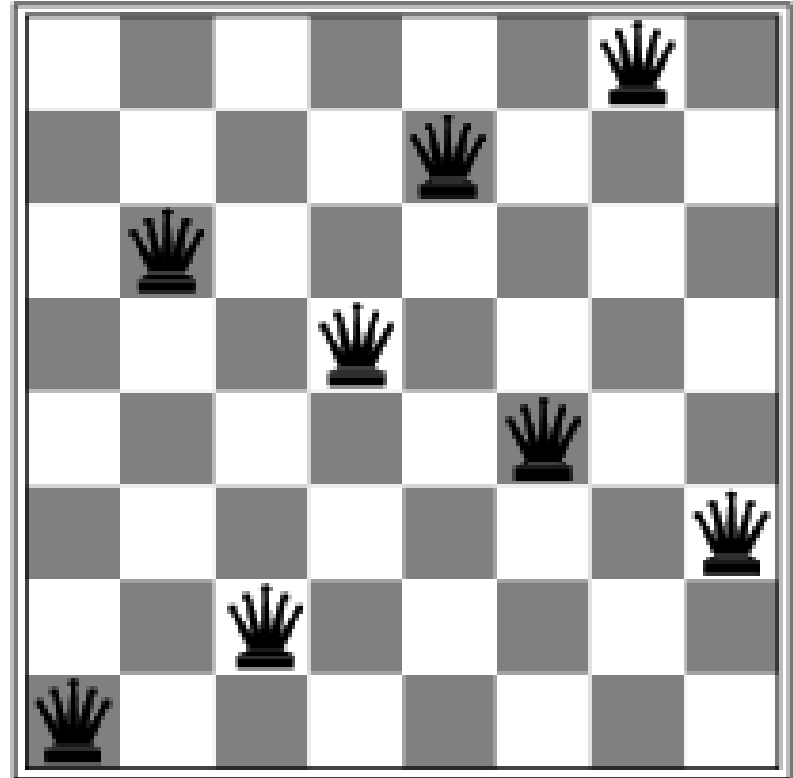
# Hill-climbing search: 8-queens problem

Each number indicates  $h$  if we move a queen in its corresponding column

|    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|
| 18 | 12 | 14 | 13 | 13 | 12 | 14 | 14 |
| 14 | 16 | 13 | 15 | 12 | 14 | 12 | 16 |
| 14 | 12 | 18 | 13 | 15 | 12 | 14 | 14 |
| 15 | 14 | 14 | ♙  | 13 | 16 | 13 | 16 |
| ♙  | 14 | 17 | 15 | ♙  | 14 | 16 | 16 |
| 17 | ♙  | 16 | 18 | 15 | ♙  | 15 | ♙  |
| 18 | 14 | ♙  | 15 | 15 | 14 | ♙  | 16 |
| 14 | 14 | 13 | 17 | 12 | 14 | 12 | 18 |

- $h$  = number of pairs of queens that are attacking each other, either directly or indirectly ( $h = 17$  for the above state)

# Hill-climbing search: 8-queens problem

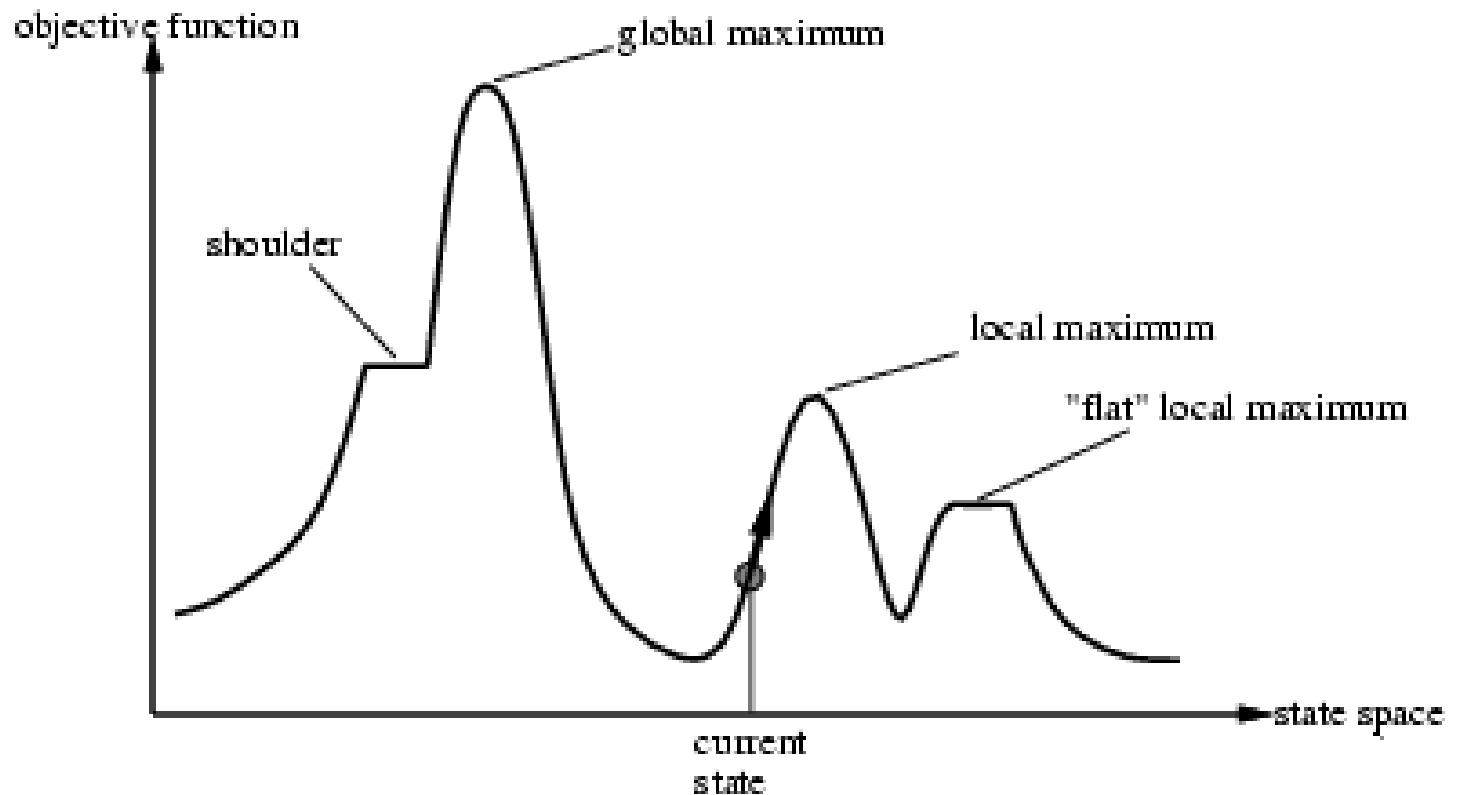


- A local minimum with  $h = 1$

(what can you do to get out of this local minima?)

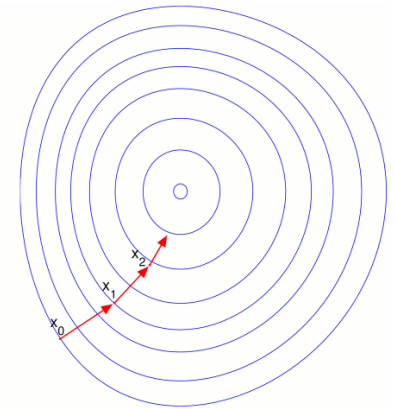
# Hill-climbing Difficulties

- Problem: depending on initial state, can get stuck in local maxima





# Gradient Descent



- Assume we have some cost-function:  $\mathcal{C}(x_1, \dots, x_n)$   
and we want minimize over continuous variables  $x_1, x_2, \dots, x_n$

1. Compute the *gradient* :  $\frac{\partial}{\partial x_i} \mathcal{C}(x_1, \dots, x_n) \quad \forall i$

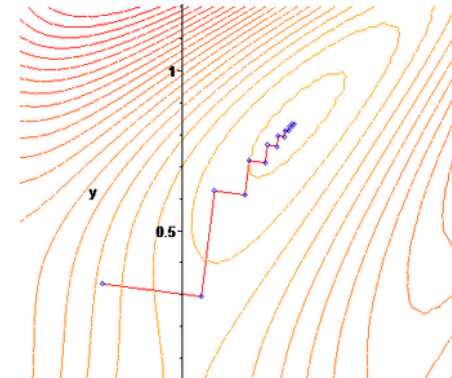
2. Take a small step downhill in the direction of the gradient:

$$x_i \rightarrow x'_i = x_i - \lambda \frac{\partial}{\partial x_i} \mathcal{C}(x_1, \dots, x_n) \quad \forall i$$

3. Check if  $\mathcal{C}(x_1, \dots, x'_i, \dots, x_n) < \mathcal{C}(x_1, \dots, x_i, \dots, x_n)$

4. If true then accept move, if not reject.

5. Repeat.



# Line Search

- In GD you need to choose a step-size.
- Line search picks a direction,  $v$ , (say the gradient direction) and searches along that direction for the optimal step:

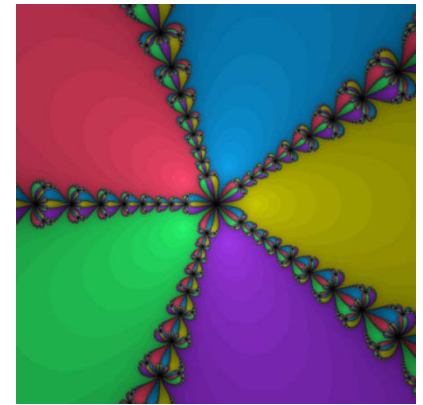
$$\eta^* = \operatorname{argmin} C(x_t + \eta v_t)$$

- Repeated doubling can be used to effectively search for the optimal step:

$$\eta \rightarrow 2\eta \rightarrow 4\eta \rightarrow 8\eta \quad (\text{until cost increases})$$

- There are many methods to pick search direction  $v$ .  
Very good method is “conjugate gradients”.

# Newton's Method



Basins of attraction for  $x^5 - 1 = 0$ ;  
darker means more iterations to converge.

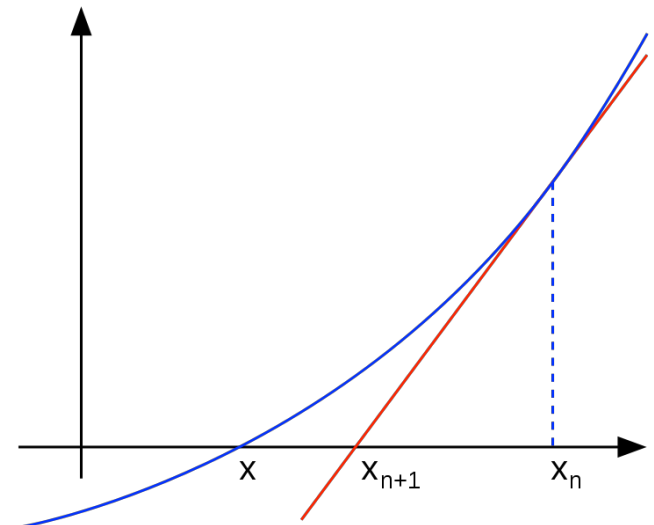
- Want to find the roots of  $f(x)$ .
- To do that, we compute the tangent at  $x_n$  and compute where it crosses the x-axis.

$$\nabla f(x_n) = \frac{0 - f(x_n)}{x_{n+1} - x_n} \Rightarrow x_{n+1} = x_n - \frac{f(x_n)}{\nabla f(x_n)}$$

- Optimization: find roots of  $\nabla f(x_n)$

$$\nabla \nabla f(x_n) = \frac{0 - \nabla f(x_n)}{x_{n+1} - x_n} \Rightarrow x_{n+1} = x_n - \frac{\nabla f(x_n)}{[\nabla \nabla f(x_n)]}$$

- Does not always converge & sometimes unstable.
- If it converges, it converges very fast



# Simulated annealing search

- Idea: escape local maxima by allowing some "bad" moves but **gradually decrease** their frequency.
- This is like smoothing the cost landscape.

# Simulated annealing search

- Idea: escape local maxima by allowing some "bad" moves but **gradually decrease** their frequency
- 

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
  inputs: problem, a problem
           schedule, a mapping from time to "temperature"
  local variables: current, a node
                   next, a node
                   T, a "temperature" controlling prob. of downward steps

  current ← MAKE-NODE(INITIAL-STATE[problem])
  for t ← 1 to ∞ do
    T ← schedule[t]
    if T = 0 then return current
    next ← a randomly selected successor of current
     $\Delta E$  ← VALUE[next] - VALUE[current]
    if  $\Delta E > 0$  then current ← next
    else current ← next only with probability  $e^{\Delta E/T}$ 
```

# Properties of simulated annealing search

- One can prove: If  $T$  decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching 1 (however, this may take VERY long)
  - However, in any finite search space RANDOM GUESSING also will find a global optimum with probability approaching 1 .
- Widely used in VLSI layout, airline scheduling, etc.

# Tabu Search

- Almost any simple local search method, but with a memory.
- Recently visited states are added to a tabu-list and are temporarily excluded from being visited again.
- This way, the solver moves away from already explored regions and (in principle) avoids getting stuck in local minima.
- Tabu search can be added to most other local search methods to obtain a variant method that avoids recently visited states.
- Tabu-list is usually implemented as a hash table for rapid access. Can also add a LIFO queue to keep track of oldest node.
- Unit time cost per step for tabu test and tabu-list maintenance.

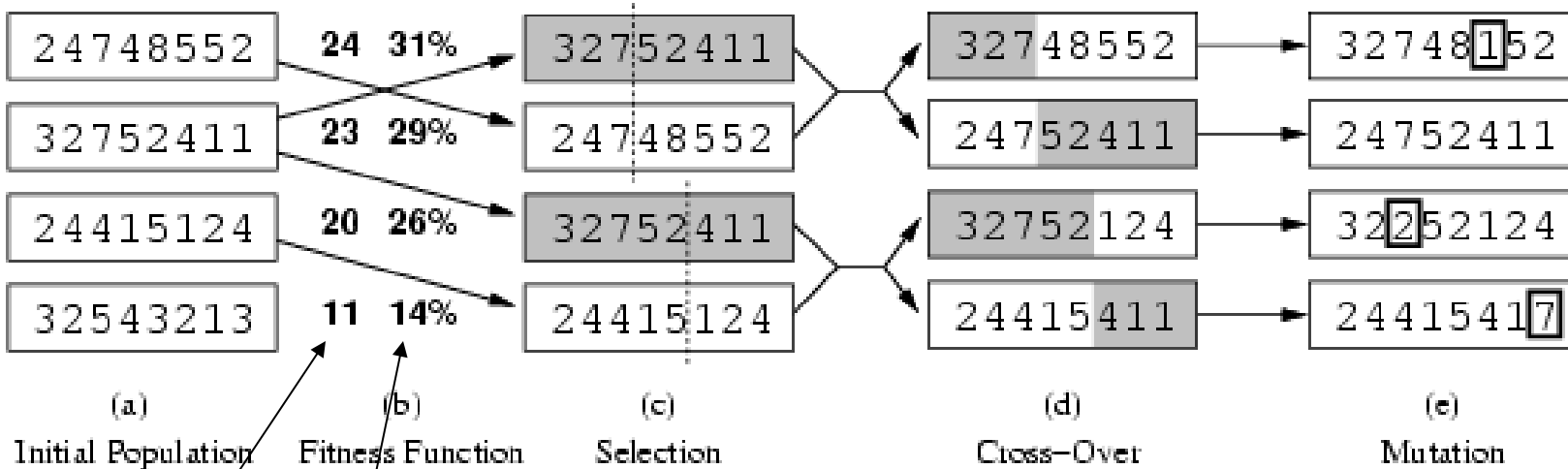
# Local beam search

- Keep track of  $k$  states rather than just one.
- Start with  $k$  randomly generated states.
- At each iteration, all the successors of all  $k$  states are generated.
- If any one is a goal state, stop; else select the  $k$  best successors from the complete list and repeat.
- Concentrates search effort in areas believed to be fruitful.
  - May lose diversity as search progresses, resulting in wasted effort.



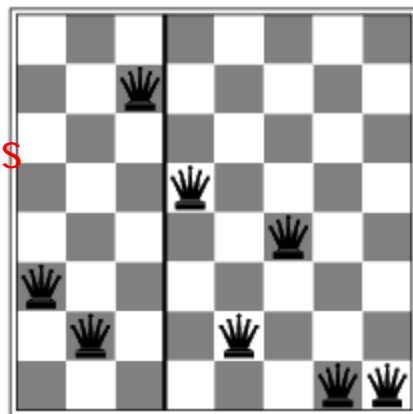
# Genetic algorithms

- A successor state is generated by combining two parent states
- Start with  $k$  randomly generated states (**population**)
- A state is represented as a string over a finite alphabet (often a string of 0s and 1s)
- Evaluation function (**fitness function**). Higher values for better states.
- Produce the next generation of states by selection, crossover, and mutation

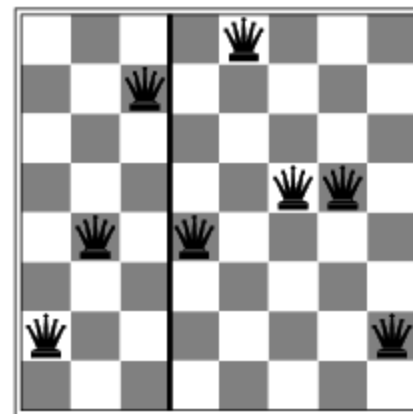


fitness:  
#non-attacking queens

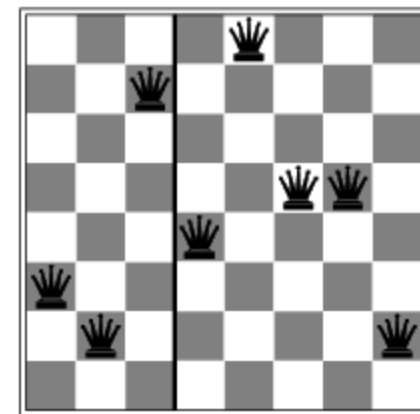
probability of being  
regenerated  
in next generation



+



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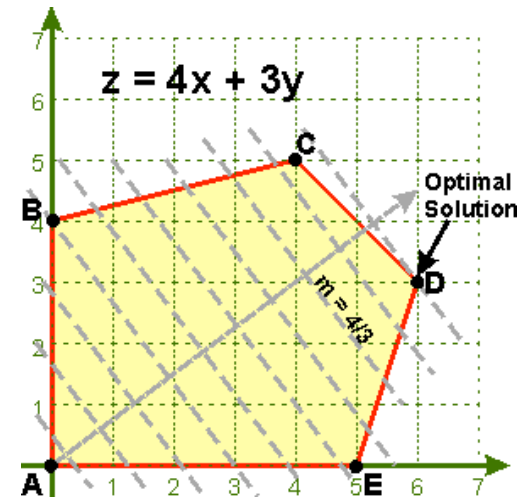


- Fitness function: number of non-attacking pairs of queens (min = 0, max =  $8 \times 7/2 = 28$ )
- $P(\text{child}) = 24/(24+23+20+11) = 31\%$
- $P(\text{child}) = 23/(24+23+20+11) = 29\%$  etc

# Linear Programming

Problems of the sort:  $\text{maximize } c^T x$   
subject to :  $Ax \leq a; Bx = b$

- Very efficient “off-the-shelves” solvers are available for LRs.
- They can solve large problems with thousands of variables.



# Linear Programming Constraints

- Maximize:  $z = c_1 x_1 + c_2 x_2 + \dots + c_n x_n$
- Primary constraints:  $x_1 \geq 0, x_2 \geq 0, \dots, x_n \geq 0$
- Additional constraints:
- $a_{i1} x_1 + a_{i2} x_2 + \dots + a_{in} x_n \leq a_i, (a_i \geq 0)$
- $a_{j1} x_1 + a_{j2} x_2 + \dots + a_{jn} x_n \geq a_j \geq 0$
- $b_{k1} x_1 + b_{k2} x_2 + \dots + b_{kn} x_n = b_k \geq 0$

# Summary

- Local search maintains a complete solution
  - Seeks to find a consistent solution (also complete)
- Path search maintains a consistent solution
  - Seeks to find a complete solution (also consistent)
- Goal of both: complete and consistent solution
  - Strategy: maintain one condition, seek other
- Local search often works well on large problems
  - Abandons optimality
  - Always has some answer available (best found so far)