Solving problems by searching

This Lecture
Read Chapters 3.1 to 3.4

Next Lecture
Read Chapter 3.5 to 3.7

(Please read lecture topic material before and after each lecture on that topic)
You will be expected to know

• State-space search
  – Definitions of a problem and of a solution
  – State-space graph

• Tree-search (don’t remember visited nodes) vs. Graph-search (do remember them)

• States vs. nodes; node implementation

• Search strategy evaluation:
  – Complete? Time/space complexity? Optimal?
  – Parameters: b, d, m
Complete architectures for intelligence?

• Search?
  – Solve the problem of what to do.

• Logic and inference?
  – Reason about what to do.
  – Encoded knowledge/"expert" systems?
  – Know what to do.

• Learning?
  – Learn what to do.

• Modern view: It’s complex & multi-faceted.
Search?
Solve the problem of what to do.

• Formulate “What to do?” as a search problem.
  – Solution to the problem tells agent what to do.

• If no solution in the current search space?
  – Formulate and solve the problem of finding a search space that does contain a solution.
  – Solve original problem in the new search space.

• Many powerful extensions to these ideas.
  – Constraint satisfaction; means-ends analysis; planning; game playing; etc.

• Human problem-solving often looks like search.
Why Search?

• We are engaged in a bigger more important problem, and we hit a search sub-problem we need to solve.
  – We need to search in order to solve it and then get back quickly to what we really wanted to do in the first place.

• To predict the result of our actions in the future.

• There are many sequences of actions, each with its utility; we wish to maximize our performance measure.

• We wish only to achieve a goal; by any means at all.
• We wish to find the best (optimal) way to achieve it.
Example: Romania

- On holiday in Romania; currently in Arad.
- Flight leaves tomorrow from Bucharest
- **Formulate goal:**
  - be in Bucharest
- **Formulate problem:**
  - **states:** various cities
  - **actions:** drive between cities or choose next city
- **Find solution:**
  - sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest
Example: Romania
Environments Types

• **Static / Dynamic**
  Previous problem was static: no attention to changes in environment

• **Observable / Partially Observable / Unobservable**
  Previous problem was observable: it knew initial state.

• **Deterministic / Stochastic**
  Previous problem was deterministic: no new percepts were necessary, we can predict the future perfectly given our actions

• **Discrete / continuous**
  Previous problem was discrete: we can enumerate all possibilities
Why not Dijkstra’s Algorithm?

• Dijkstra’s algorithm inputs the entire graph.
  – We want to search in unknown spaces.
  – Essentially, we want to combine search with exploration.
  – E.g., an autonomous rover on Mars must search an unknown space.

• D’s algorithm takes connections as given.
  – We want to search based on agent’s actions, with unknown connections.
  – E.g., a Web-crawler may not know what further connections are available on an unexplored URL before visiting it.
  – E.g., the agent may not know the result of an action before trying it.

• D’s algorithm won’t work on infinite spaces.
  – We want to search in infinite spaces.
  – E.g., the logical reasoning space is infinite.
  – E.g., the real world is essentially infinite to a human-size agent.
Example: vacuum world

- Observable, start in #5.

Solution?
Example: vacuum world

- Observable, start in #5.
  Solution?
  [Right, Suck]
Vacuum world state space graph
Example: vacuum world

- Unobservable, start in \{1,2,3,4,5,6,7,8\} e.g., Solution?
Example: vacuum world

- Unobservable, start in \{1,2,3,4,5,6,7,8\} e.g.,
  Solution?
  [Right, Suck, Left, Suck]
Problem Formulation

A problem is defined by five items:

1. **initial state** e.g., "at Arad"
2. **actions** Actions(s) = set of actions available in state s
3. **transition model** Result(s,a) = state that results from action a in state s
   (alternative: successor function) \( S(s) = \text{set of action–state pairs} \)
   e.g., \( S(\text{Arad}) = \{<\text{Arad} \rightarrow \text{Zerind, Sibiu, Timisoara}>, \ldots \} \)
4. **goal test**, e.g., \( s = \text{"at Bucharest"}, \text{Checkmate}(s) \)
5. **path cost** (additive) e.g., sum of distances, number of actions executed, etc.
   – \( c(x,a,y) \) is the **step cost**, assumed to be \( \geq 0 \), summed to yield path cost

A solution = sequence of actions leading from initial state to a goal state
Selecting a state space

• Real world is absurdly complex
  → state space must be abstracted for problem solving

• (Abstract) state ← set of real states

• (Abstract) action ← complex combination of real actions
  – e.g., "Arad → Zerind" represents a complex set of possible routes, detours, rest stops, etc.

• For guaranteed realizability, any real state "in Arad" must get to some real state "in Zerind"

• (Abstract) solution ← set of real paths that are solutions in the real world

• Each abstract action should be "easier" than the original problem
Vacuum world state space graph

- **states?** discrete: dirt and robot location
- **initial state?** any
- **actions?** *Left, Right, Suck*
- **goal test?** no dirt at all locations
- **path cost?** 1 per action
Example: 8-Queens

- **states?** - any arrangement of $n \leq 8$ queens
  - or arrangements of $n \leq 8$ queens in leftmost $n$ columns, 1 per column, such that no queen attacks any other (BETTER!!).
- **initial state?** no queens on the board
- **actions?** - add queen to any empty square
  - or add queen to leftmost empty square, such that it is not attacked by other queens (BETTER!!).
- **goal test?** 8 queens on the board, none attacked.
- **path cost?** 1 per move (not really relevant)
Example: robotic assembly

- **states**: real-valued coordinates of robot joint angles parts of the object to be assembled
- **initial state**: rest configuration
- **actions**: continuous motions of robot joints
- **goal test**: complete assembly
- **path cost**: time to execute+energy used
Example: The 8-puzzle

- states?
- initial state?
- actions?
- goal test?
- path cost?

Try yourselves
Example: The 8-puzzle

- **states?** locations of tiles
- **initial state?** given
- **actions?** move blank left, right, up, down
- **goal test?** goal state (given)
- **path cost?** 1 per move

[Note: optimal solution of $n$-Puzzle family is NP-hard]
Tree search algorithms

• Basic idea:
  – Exploration of state space by generating successors of already-explored states (a.k.a.~expanding states).
  – Every generated state is evaluated: *is it a goal state?*
Tree search example
Tree search example
Note that we come back to Arad often, wasting time & work

We will visit the same node often, wasting time & work

function TREE-SEARCH(problem, strategy) returns a solution, or failure
initialize the search tree using the initial state of problem
loop do
  if there are no candidates for expansion then return failure
  choose a leaf node for expansion according to strategy
  if the node contains a goal state then return the corresponding solution
  else expand the node and add the resulting nodes to the search tree
Repeated states

- Failure to detect repeated states can turn a linear problem into an exponential one!
- Test is often implemented as a hash table.
Solutions to Repeated States

• Graph search
  – never generate a state generated before
    • must keep track of all possible states (uses a lot of memory)
    • e.g., 8-puzzle problem, we have $9! = 362,880$ states
    • approximation for DFS/DLS: only avoid states in its (limited) memory: avoid infinite loops by checking path back to root.
  – “visited?” test usually implemented as a hash table

State Space

Example of a Search Tree

faster, but memory inefficient
Implementation: states vs. nodes

- A **state** is a (representation of) a physical configuration.

- A **node** is a data structure constituting part of a search tree contains info such as: state, parent node, action, path cost $g(x)$, depth.

- The **Expand function** creates new nodes, filling in the various fields and using the **SuccessorFn** of the problem to create the corresponding states.
Search strategies

• A search strategy is defined by picking the order of node expansion

• Strategies are evaluated along the following dimensions:
  – completeness: does it always find a solution if one exists?
  – time complexity: number of nodes generated
  – space complexity: maximum number of nodes in memory
  – optimality: does it always find a least-cost solution?

• Time and space complexity are measured in terms of
  – $b$: maximum branching factor of the search tree
  – $d$: depth of the least-cost solution
  – $m$: maximum depth of the state space (may be $\infty$)
Summary

• Generate the search space by applying actions to the initial state and all further resulting states.

• Problem: initial state, actions, transition model, goal test, step/path cost

• Solution: sequence of actions to goal

• Tree-search (don’t remember visited nodes) vs. Graph-search (do remember them)

• Search strategy evaluation: b, d, m
  – Complete? Time? Space? Optimal?