First-Order Logic
Semantics (& no inference, but including Unification)

First Lecture Today (Tue 28 Jun)
Review Chapters 8.3-8.5,
Read 9.1-9.2 (optional: 9.5)

Second Lecture Today (Tue 28 Jun)
Read Chapters 13, 14.1-14.5

Next Lecture (Thu 30 Jun)
Read Chapters 3.1-3.4, 18.6.1-2, 20.3.1

(Please read lecture topic material before and after each lecture on that topic)
You will be expected to know

- Semantics, Worlds, and Interpretations
- Nested quantifiers
  - Difference between “\( \forall x \exists y P(x, y) \)” and “\( \exists x \forall y P(x, y) \)”
  - \( \forall x \{ \exists y \text{ Person}(x) \Rightarrow \text{Likes}(x, y) \} \)
    - = Every person (every person \( x \)) likes something (likes some \( y \)).
    - Can be a different \( y \) for each \( x \) (think about variable scope!)
  - \( \exists x \{ \forall y \text{ Person}(x) \land \text{Likes}(x, y) \} \)
    - = There is some person (some \( x \)) that likes everything (every \( y \)).
    - Must be the same \( x \) for every \( y \) (think about variable scope!)
- Translate simple English sentences to FOPC and back
  - \( \forall x \exists y \text{ Likes}(x, y) \) = “Every person has some person that they like.”
  - \( \exists x \forall y \text{ Likes}(x, y) \) = “There is some person who likes every person.”
  - Technically, there should be \( \text{Person}(x) \land \text{Person}(y) \) predicates above
- Unification: Given two FOL terms containing variables
  - Find the most general unifier if one exists.
  - Else, explain why no unification is possible.
  - See Section 9.2 and Figure 9.1 in your textbook.
Outline

- Review: $\text{KB} \models S$ is equivalent to $\models (\text{KB} \Rightarrow S)$
  - So what does $\{\} \models S$ mean?

- Review: Follows, Entails, Derives
  - Follows: “Is it the case?”
  - Entails: “Is it true?”
  - Derives: “Is it provable?”

- Semantics of FOL (FOPC)
  - Model, Interpretation

- Unification
FOL (or FOPC) Ontology:
What kind of things exist in the world?
What do we need to describe and reason about?
Objects --- with their relations, functions, predicates, properties, and general rules.
Review: $KB \models S$ means $\models (KB \Rightarrow S)$

- $KB \models S$ is read “$KB$ entails $S$.”
  - Means “$S$ is true in every world (model) in which $KB$ is true.”
  - Means “In the world, $S$ follows from $KB$.”

- $KB \models S$ is equivalent to $\models (KB \Rightarrow S)$
  - Means “$(KB \Rightarrow S)$ is true in every world (i.e., is valid).”

- And so: $\{\} \models S$ is equivalent to $\models (\{\} \Rightarrow S)$

- So what does $(\{\} \Rightarrow S)$ mean?
  - Means “$True$ implies $S$.”
  - Means “$S$ is valid.”
  - In Horn form, means “$S$ is a fact.” p. 256 (3rd ed.; p. 281, 2nd ed.)

- Why does $\{\} \models S$ mean True here, but means False in resolution proofs?
Review: \((\text{True} \Rightarrow \text{S})\) means “\text{S} is a fact.”

- By convention,
  - The null conjunct is “syntactic sugar” for True.
  - The null disjunct is “syntactic sugar” for False.
  - Each is assigned the truth value of its identity element.
    - For conjuncts, True is the identity: \((A \land \text{True}) \equiv A\)
    - For disjuncts, False is the identity: \((A \lor \text{False}) \equiv A\)

- A KB is the conjunction of all of its sentences.
  - So in the expression: \(\{\} \models \text{S}\)
    - We see that \(\{\}\) is the null conjunct and means True.
    - The expression means “\text{S} is true in every world where True is true.”
      - I.e., “\text{S} is valid.”
    - Better way to think of it: \(\{\}\) does not exclude any worlds (models).

- In Conjunctive Normal Form each clause is a disjunct.
  - So in, say, \(\text{KB} = \{ (P \land Q) \lor (\neg Q \land R) \lor (X \lor Y) \lor (\neg Z) \}\)
    - We see that \((\_\_\_)\) is the null disjunct and means False.
Side Trip: Functions AND, OR, and null values
(Note: These are “syntactic sugar” in logic.)

function AND(arglist) returns a truth-value
    return ANDOR(arglist, True)

function OR(arglist) returns a truth-value
    return ANDOR(arglist, False)

function ANDOR(arglist, nullvalue) returns a truth-value
    /* nullvalue is the identity element for the caller. */
    if (arglist = {}) then return nullvalue
    if ( FIRST(arglist) = NOT(nullvalue) ) then return NOT(nullvalue)
    return ANDOR( REST(arglist), nullvalue )
Side Trip: We only need one logical connective. (Note: AND, OR, NOT are “syntactic sugar” in logic.)

Both NAND and NOR are logically complete.

- NAND is also called the “Sheffer stroke”
- NOR is also called “Pierce’s arrow”

\[(\text{NOT } A) = (\text{NAND } A \text{ TRUE}) = (\text{NOR } A \text{ FALSE})\]

\[(\text{AND } A \text{ B}) = (\text{NAND } \text{ TRUE} (\text{NAND } A \text{ B})) = (\text{NOR } (\text{NOR } A \text{ FALSE}) (\text{NOR } B \text{ FALSE}))\]

\[(\text{OR } A \text{ B}) = (\text{NAND } (\text{NAND } A \text{ TRUE}) (\text{NAND } B \text{ TRUE})) = (\text{NOR } \text{ FALSE} (\text{NOR } A \text{ B}))\]

This fact is exploited by, e.g., VLSI semiconductor fabrication, which often provide a single NAND/NOR gate for efficiency.
If KB is true in the real world, then any sentence \( \alpha \) entailed by KB and any sentence \( \alpha \) derived from KB by a sound inference procedure is also true in the real world.
Schematic Example: Follows, Entails, and Derives

**Inference**

- “Mary is Sue’s sister and Amy is Sue’s daughter.”
- “An aunt is a sister of a parent.”

**Derives**

- Derives
- Is it provable?

**Entails**

- Entails
- Is it true?

**Follows**

- Follows
- Is it the case?

**World**

- Mary
- Sister
- Sue
- Daughter
- Amy
- Mary
- Aunt
- Amy

Need also to know: Daughter(x,y) => Parent(y,x)
Review: Models (and in FOL, Interpretations)

- **Models** are formal worlds within which truth can be evaluated.
- **Interpretations** map symbols in the logic to the world:
  - Constant symbols in the logic map to objects in the world.
  - n-ary functions/predicates map to n-ary functions/predicates in the world.

We say *m is a model given an interpretation i* of a sentence $\alpha$ if and only if $\alpha$ is true in the world $m$ under the mapping $i$.

Your job, as knowledge engineers, is to ensure that *only* your intended worlds and interpretations make your KB true.

In the circuit world, you Tell it: “(1 not= 0)”.

In the biology world, you Tell it: “$\forall x \ (\text{Cat}(x) \Rightarrow \text{Mammal}(x))$”

If you fail to Tell it these facts, then it will make stupid inferences that you will have to come back later and debug, to fix your KB.

You know *all* these things. It doesn’t know any of them. Stupid...
We say \textit{m is a model given an interpretation i} of a sentence \( \alpha \) if and only if \( \alpha \) is true in the world \( m \) under the mapping \( i \).

\( M(\alpha) \) is the set of all models of \( \alpha \).

Then \( KB \models \alpha \) iff \( M(KB) \subseteq M(\alpha) \).

E.g. \( KB = \) “Mary is Sue’s sister and Amy is Sue’s daughter.”

\( \alpha = \) “Mary is Amy’s aunt.” (Must Tell it about mothers/daughters)

Think of \( KB \) and \( \alpha \) as constraints, and models as states.

\( M(KB) \) are the solutions to \( KB \) and \( M(\alpha) \) the solutions to \( \alpha \).

Then, \( KB \models \alpha \), i.e., \( \models (KB \Rightarrow \alpha) \),
when all solutions to \( KB \) are also solutions to \( \alpha \).
Review: Semantics of Worlds

• The world consists of objects that have properties.
  - There are relations and functions between these objects
  - Objects in the world, individuals: people, houses, numbers, colors, baseball games, wars, centuries
    • Clock A, John, 7, the-house in the corner, Tel-Aviv, Ball43
  - Functions on individuals:
    • father-of, best friend, third inning of, one more than
  - Relations:
    • brother-of, bigger than, inside, part-of, has color, occurred after
  - Properties (a relation of arity 1):
    • red, round, bogus, prime, multistoried, beautiful
Semantics: Interpretation

- An **interpretation** of a sentence (wff) is an assignment that maps
  - Object constant symbols to objects in the world,
  - n-ary function symbols to n-ary functions in the world,
  - n-ary relation symbols to n-ary relations in the world

- Given an interpretation, an atomic sentence has the value “true” if it denotes a relation that holds for those individuals denoted in the terms. Otherwise it has the value “false.”
  - Example: Kinship world:
    - Symbols = Ann, Bill, Sue, Married, Parent, Child, Sibling, ...
    - World consists of individuals in relations:
      - Married(Ann,Bill) is false, Parent(Bill,Sue) is true, ...

- **Your job, as a Knowledge Engineer, is to construct KB so it is true *exactly* for your world and intended interpretation.**
Truth in first-order logic

- Sentences are true with respect to a model and an interpretation.
- Model contains objects (domain elements) and relations among them.
- Interpretation specifies referents for:
  - constant symbols → objects
  - predicate symbols → relations
  - function symbols → functional relations
- An atomic sentence \( \text{predicate}(\text{term}_1, \ldots, \text{term}_n) \) is true iff the objects referred to by \( \text{term}_1, \ldots, \text{term}_n \) are in the relation referred to by \( \text{predicate} \).
Semantics: Models and Definitions

- An interpretation and possible world **satisfies** a wff (sentence) if the wff has the value “true” under that interpretation in that possible world.
- Model: A domain and an interpretation that satisfies a wff is a **model** of that wff.
- Validity: Any wff that has the value “true” in all possible worlds and under all interpretations is **valid**.
- Any wff that does not have a model under any interpretation is inconsistent or **unsatisfiable**.
- Any wff that is true in at least one possible world under at least one interpretation is **satisfiable**.
- If a wff w has a value true under all the models of a set of sentences KB then KB logically **entails** w.
Models for FOL: Example

An interpretation maps all symbols in KB onto matching symbols in the domain in all possible ways.

An interpretation maps all symbols in KB onto matching symbols in a possible world. All possible interpretations gives a combinatorial explosion of mappings. Your job, as a Knowledge Engineer, is to write the axioms in KB so they are satisfied only under the intended interpretation in your own real world.
Summary of FOL Semantics

- A well-formed formula (“wff”) FOL is true or false with respect to a world and an interpretation (a model).

- The world has objects, relations, functions, and predicates.

- The interpretation maps symbols in the logic to the world.

- The wff is true if and only if (iff) its assertion holds among the objects in the world under the mapping by the interpretation.

- Your job, as a Knowledge Engineer, is to write sufficient KB axioms that ensure that KB is true in your own real world under your own intended interpretation.
  - The KB axioms must rule out other worlds and interpretations.
Summary of Conversion to CNF in FOL

• In order to cover less material with more thoroughness, I no longer cover conversion to CNF and resolution in FOL.
  – Intuitions from Propositional Logic are a good base for FOL.

• The major different points are these:
  • Existential quantification is replaced by a “Skolem constant”
    – For example,
      \[ \exists x \text{ Lives}_\text{in}(John, \text{Castle}(x)) \]  “John lives in somebody’s castle.”
      is replaced by \text{Lives}_\text{in}(John, \text{Castle}(Symbol_97))
      “John lives in Symbol_97’s castle”
  • So, all remaining variables must be universally quantified
    – For example, \[ \forall x \text{ King}(x) \Rightarrow \text{Person}(x) \] becomes
      \[ (\neg \text{King}(x_{73}) \lor \text{Person}(x_{73})) \]
    – All variables always given a different name (“standardizing apart”)

• Resolution on predicate terms after unification (substitution)
  – For example, “\((\neg \text{King}(x_{73}) \text{ Person}(x_{73}))\)” with “King(John)”
    yields “\text{Person}(John)” because of the substitution \(x_{73}/\text{John}\)
**Unification**

- Recall: Subst(θ, p) = result of substituting θ into sentence p

- Unify algorithm: takes 2 sentences p and q and returns a unifier if one exists

  \[
  \text{Unify}(p,q) = \theta \quad \text{where} \quad \text{Subst}(\theta, p) = \text{Subst}(\theta, q)
  \]

- Example:
  
  \[
  p = \text{Knows(John,x)} \\
  q = \text{Knows(John, Jane)}
  \]

  \[
  \text{Unify}(p,q) = \{x/\text{Jane}\} 
  \]
Unification examples

- simple example: query = Knows(John,x), i.e., who does John know?

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knows(John,x)</td>
<td>Knows(John,Jane)</td>
<td>{x/Jane}</td>
</tr>
<tr>
<td>Knows(John,x)</td>
<td>Knows(y,OJ)</td>
<td>{x/OJ,y/John}</td>
</tr>
<tr>
<td>Knows(John,x)</td>
<td>Knows(y,Mother(y))</td>
<td>{y/John,x/Mother(John)}</td>
</tr>
<tr>
<td>Knows(John,x)</td>
<td>Knows(x,OJ)</td>
<td>{fail}</td>
</tr>
</tbody>
</table>

- Last unification fails: only because x can’t take values John and OJ at the same time
  - But we know that if John knows x, and everyone (x) knows OJ, we should be able to infer that John knows OJ

- Problem is due to use of same variable x in both sentences

- Simple solution: Standardizing apart eliminates overlap of variables, e.g., Knows(z,OJ)
Unification

- To unify $\text{Knows}(John,x)$ and $\text{Knows}(y,z)$,

$$\theta = \{y/John, x/z\} \text{ or } \theta = \{y/John, x/John, z/John\}$$

- The first unifier is more general than the second.

- There is a single most general unifier (MGU) that is unique up to renaming of variables.

$$\text{MGU} = \{y/John, x/z\}$$

- General algorithm in Figure 9.1 in the text
Unification Algorithm

function UNIFY\( (x, y, \theta) \) returns a substitution to make \( x \) and \( y \) identical inputs: \( x \), a variable, constant, list, or compound expression \( y \), a variable, constant, list, or compound expression \( \theta \), the substitution built up so far (optional, defaults to empty)

if \( \theta = \text{failure} \) then return failure
else if \( x = y \) then return \( \theta \)
else if VARIABLE\( ?(x) \) then return UNIFY-VAR\( (x, y, \theta) \)
else if VARIABLE\( ?(y) \) then return UNIFY-VAR\( (y, x, \theta) \)
else if COMPOUND\( ?(x) \) and COMPOUND\( ?(y) \) then
  return UNIFY\( (x.\text{ARGS}, y.\text{ARGS}, \text{UNIFY}(x.\text{OP}, y.\text{OP}, \theta)) \)
else if LIST\( ?(x) \) and LIST\( ?(y) \) then
  return UNIFY\( (x.\text{REST}, y.\text{REST}, \text{UNIFY}(x.\text{FIRST}, y.\text{FIRST}, \theta)) \)
else return failure

function UNIFY-VAR\( (\text{var}, x, \theta) \) returns a substitution

if \{ var/val \} \in \theta \) then return UNIFY\( (\text{val}, x, \theta) \)
else if \{ x/\text{val} \} \in \theta \) then return UNIFY\( (\text{var}, \text{val}, \theta) \)
else if OCCUR-CHECK\( ?(\text{var}, x) \) then return failure
else return add \{ \text{var}/\text{x} \} \) to \( \theta \)

Figure 9.1 The unification algorithm. The algorithm works by comparing the structures of the inputs, element by element. The substitution \( \theta \) that is the argument to UNIFY is built up along the way and is used to make sure that later comparisons are consistent with bindings that were established earlier. In a compound expression such as \( F(A, B) \), the \( \text{Op} \) field picks out the function symbol \( F \) and the \( \text{ARGS} \) field picks out the argument list \( (A, B) \).
Unification Algorithm

function UNIFY(x, y, θ) returns a substitution to make x and y identical
inputs: x, a variable, constant, list, or compound expression
y, a variable, constant, list, or compound expression
θ, the substitution built up so far (optional, defaults to empty)

if θ = failure then return failure
else if x = y then return θ
else if VARIABLE?(x) then return UNIFY-VAR(x, y, θ)
else if VARIABLE?(y) then return UNIFY-VAR(y, x, θ)
else if COMPOUND?(x) and COMPOUND?(y) then
  return UNIFY(x.ARGS, y.ARGS, UNIFY(x.OP, y.OP, θ))
else if LIST?(x) and LIST?(y) then
  return UNIFY(x.REST, y.REST, UNIFY(x.FIRST, y.FIRST, θ))
else return failure

function UNIFY-VAR(var, x, θ) returns a substitution

if \{var/val\} ⊆ θ then return UNIFY(val, x, θ)
else if \{x/val\} ⊆ θ then return UNIFY(var, val, θ)
else if OCCUR-CHECK?(var, x) then return failure
else return add \{var/x\} to θ

Figure 9.1 The unification algorithm. The algorithm works by comparing the structures of the inputs, element by element. The substitution θ that is the argument to UNIFY is built up along the way and is used to make sure that later comparisons are consistent with bindings that were established earlier. In a compound expression such as $F(A, B)$, the OP field picks out the function symbol $F$ and the ARGS field picks out the argument list $(A, B)$. 

If we have failed or succeeded, then fail or succeed.
Unification Algorithm

function UNIFY\(x, y, \theta\) returns a substitution to make \(x\) and \(y\) identical
inputs: \(x\), a variable, constant, list, or compound expression
\(y\), a variable, constant, list, or compound expression
\(\theta\), the substitution built up so far (optional, defaults to empty)

if \(\theta = \text{failure}\) then return \(\text{failure}\)
else if \(x = y\) then return \(\theta\)
else if VARIABLE\(?\)(x) then return UNIFY-VAR\(x, y, \theta\)
else if VARIABLE\(?\)(y) then return UNIFY-VAR\(y, x, \theta\)
else if COMPOUND\(?\)(x) and COMPOUND\(?\)(y) then
    return UNIFY\(x.\text{ARGS}, y.\text{ARGS}, UNIFY(x.\text{OP}, y.\text{OP}, \theta)\)
else if LIST\(?\)(x) and LIST\(?\)(y) then
    return UNIFY\(x.\text{REST}, y.\text{REST}, UNIFY(x.\text{FIRST}, y.\text{FIRST}, \theta)\)
else return \(\text{failure}\)

function UNIFY-VAR\(\text{var}, x, \theta\) returns a substitution

if \(\{\text{var/val}\} \subseteq \theta\) then return UNIFY\(\text{val}, x, \theta\)
else if \(\{\text{x/val}\} \subseteq \theta\) then return UNIFY\(\text{var}, \text{val}, \theta\)
else if OCCUR-CHECK\(?\)(\text{var}, x) then return \(\text{failure}\)
else return add \(\{\text{var/x}\}\) to \(\theta\)

Figure 9.1 The unification algorithm. The algorithm works by comparing the structures of the inputs, element by element. The substitution \(\theta\) that is the argument to UNIFY is built up along the way and is used to make sure that later comparisons are consistent with bindings that were established earlier. In a compound expression such as \(F(A, B)\), the Op field picks out the function symbol \(F\) and the ARGS field picks out the argument list \((A, B)\).
Unification Algorithm

function UNIFY(x, y, θ) returns a substitution to make x and y identical
inputs: x, a variable, constant, list, or compound expression
         y, a variable, constant, list, or compound expression
         θ, the substitution built up so far (optional, defaults to empty)

if θ = failure then return failure
else if x = y then return θ
else if VARIABLE?(x) then return UNIFY-VAR(x, y, θ)
else if VARIABLE?(y) then return UNIFY-VAR(y, x, θ)
else if COMPOUND?(x) and COMPOUND?(y) then
    return UNIFY(x.ARGS, y.ARGS, UNIFY(x.OP, y.OP, θ))
else if LIST?(x) and LIST?(y) then
    return UNIFY(x.REST, y.REST, UNIFY(x.FIRST, y.FIRST, θ))
else return failure

function UNIFY-VAR(var, x, θ) returns a substitution
if {var/val} ∈ θ then return UNIFY(val, x, θ)
else if {x/val} ∈ θ then return UNIFY(var, val, θ)
else if OCCUR-CHECK?(var, x) then return failure
else return add {var/x} to θ

If we already have bound variable var to a value, try to continue on that basis.

There is an implicit assumption that “{var/val} ∈ θ”, if it succeeds, binds val to the value that allowed it to succeed, that were established earlier. In a compound expression such as F(A, B), the Op field picks out the function symbol F and the ARGS field picks out the argument list (A, B).
Unification Algorithm

function UNIFY(x, y, θ) returns a substitution to make x and y identical
inputs: x, a variable, constant, list, or compound expression
         y, a variable, constant, list, or compound expression
         θ, the substitution built up so far (optional, defaults to empty)

if θ = failure then return failure
else if x = y then return θ
else if VARIABLE?(x) then return UNIFY-VAR(x, y, θ)
else if VARIABLE?(y) then return UNIFY-VAR(y, x, θ)
else if COMPOUND?(x) and COMPOUND?(y) then
    return UNIFY(x.ARGs, y.ARGs, UNIFY(x.OP, y.OP, θ))
else if LIST?(x) and LIST?(y) then
    return UNIFY(x.REST, y.REST, UNIFY(x.FIRST, y.FIRST, θ))
else return failure

function UNIFY-VAR(var, x, θ) returns a substitution

if \{var/val\} ∈ θ then return UNIFY(val, x, θ)
else if \{x/val\} ∈ θ then return UNIFY(var, val, θ)
else if OCCUR-CHECK?(var, x) then return failure
else return add \{var/x\} to θ

If we already have bound x to a value, try to continue on that basis.

Figure 9.1 The unification algorithm. The algorithm works by comparing the structures of the inputs, element by element. The substitution θ that is the argument to UNIFY is built up along the way and is used to make sure that later comparisons are consistent with bindings that were established earlier. In a compound expression such as F(A, B), the Op field picks out the function symbol F and the ARGs field picks out the argument list (A, B).
Unification Algorithm

function UNIFY(x, y, \theta) returns a substitution to make x and y identical
inputs: x, a variable, constant, list, or compound expression
y, a variable, constant, list, or compound expression
\theta, the substitution built up so far (optional, defaults to empty)

if \theta = failure then return failure
else if x = y then return \theta
else if VARIABLE?(x) then return UNIFY-VAR(x, y, \theta)
else if VARIABLE?(y) then return UNIFY-VAR(y, x, \theta)
else if COMPOUND?(x) and COMPOUND?(y) then
  return UNIFY(x.ARG, y.ARG, UNIFY(x.OP, y.OP, \theta))
else if LIST?(x) and LIST?(y) then
  return UNIFY(x.REST, y.REST, UNIFY(x.FIRST, y.FIRST, \theta))
else return failure

function UNIFY-VAR(var, x, \theta) returns a substitution

if \{var/val\} \subseteq \theta then return UNIFY(val, x, \theta)
else if \{x/val\} \subseteq \theta then return UNIFY(var, val, \theta)
else if OCCUR-CHECK?(var, x) then return failure
else return add \{var/x\} to \theta

If var occurs anywhere within x, then no substitution will succeed.

Figure 9.1 The unification algorithm. The algorithm works by comparing the structures of the inputs, element by element. The substitution \theta that is the argument to UNIFY is built up along the way and is used to make sure that later comparisons are consistent with bindings that were established earlier. In a compound expression such as F(A, B), the Op field picks out the function symbol F and the ARGS field picks out the argument list (A, B).
Unification Algorithm

function UNIFY($x$, $y$, $\theta$) returns a substitution to make $x$ and $y$ identical
inputs: $x$, a variable, constant, list, or compound expression
$y$, a variable, constant, list, or compound expression
$\theta$, the substitution built up so far (optional, defaults to empty)

if $\theta =$ failure then return failure
else if $x = y$ then return $\theta$
else if VARIABLE?($x$) then return UNIFY-VAR($x$, $y$, $\theta$)
else if VARIABLE?($y$) then return UNIFY-VAR($y$, $x$, $\theta$)
else if COMPOUND?($x$) and COMPOUND?($y$) then
    return UNIFY($x$ARGS, $y$ARGS, UNIFY($x$OP, $y$OP, $\theta$))
else if LIST?($x$) and LIST?($y$) then
    return UNIFY($x$REST, $y$REST, UNIFY($x$FIRST, $y$FIRST, $\theta$))
else return failure

function UNIFY-VAR($var$, $x$, $\theta$) returns a substitution

if $\{var/val\} \in \theta$ then return UNIFY($val$, $x$, $\theta$)
else if $\{x/val\} \in \theta$ then return UNIFY($var$, $val$, $\theta$)
else if OCCUR-CHECK?($var$, $x$) then return failure
else return add $\{var/x\}$ to $\theta$

Figure 9.1 The unification algorithm. The algorithm works by comparing the structures of the inputs, element by element. The substitution $\theta$ that is the argument to UNIFY is built up along the way and is used to make sure that later comparisons are consistent with bindings that were established earlier. In a compound expression such as $F(A, B)$, the Op field picks out the function symbol $F$ and the ARGS field picks out the argument list ($A, B$).
The unification algorithm. The algorithm works by comparing the structures of the inputs, element by element. The substitution \( \theta \) that is the argument to UNIFY is built up along the way and is used to make sure that later comparisons are consistent with bindings that were established earlier. In a compound expression such as \( F(A, B) \), the Op field picks out the function symbol \( F \) and the ARGS field picks out the argument list \( (A, B) \).
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y, a variable, constant, list, or compound expression
\theta, the substitution built up so far (optional, defaults to empty)

if \theta = failure then return failure
else if x = y then return \theta
else if VARIABLE?(x) then return UNIFY-VAR(x, y, \theta)
else if VARIABLE?(y) then return UNIFY-VAR(y, x, \theta)
else if COMPOUND?(x) and COMPOUND?(y) then
    return UNIFY(x.ARGS, y.ARGS, UNIFY(x.OP, y.OP, \theta))
else if LIST?(x) and LIST?(y) then
    return UNIFY(x.REST, y.REST, UNIFY(x.FIRST, y.FIRST, \theta))
else return failure

function UNIFY-VAR(var, x, \theta) returns a substitution

if \{var/val\} \subseteq \theta then return UNIFY(val, x, \theta)
else if \{x/val\} \subseteq \theta then return UNIFY(var, val, \theta)
else if OCCUR-CHECK?(var, x) then return failure
else return add \{var/x\} to \theta

Figure 9.1 The unification algorithm. The algorithm works by comparing the structures of the inputs, element by element. The substitution \theta that is the argument to UNIFY is built up along the way and is used to make sure that later comparisons are consistent with bindings that were established earlier. In a compound expression such as F(A, B), the Op field picks out the function symbol F and the ARGS field picks out the argument list (A, B).
Unification Algorithm

```
function UNIFY(x, y, θ) returns a substitution to make x and y identical
inputs: x, a variable, constant, list, or compound expression
        y, a variable, constant, list, or compound expression
        θ, the substitution built up so far (optional, defaults to empty)

if θ = failure then return failure
else if x = y then return θ
else if VARIABLE?(x) then return UNIFY-VAR(x, y, θ)
else if VARIABLE?(y) then return UNIFY-VAR(y, x, θ)
else if COMPOUND?(x) and COMPOUND?(y) then
    return UNIFY(x ARGS, y ARGS, UNIFY(x OP, y OP, θ))
else if LIST?(x) and LIST?(y) then
    return UNIFY(x REST, y REST, UNIFY(x FIRST, y FIRST, θ))
else return failure
```

```
function UNIFY-VAR(var, x, θ) returns a substitution

if {var/val} ∈ θ then return UNIFY(val, x, θ)
else if {x/val} ∈ θ then return UNIFY(var, val, θ)
else if OCCUR-CHECK?(var, x) then return failure
else return add {var/x} to θ
```

Figure 9.1 The unification algorithm. The algorithm works by comparing the structures of the inputs, element by element. The substitution θ that is the argument to UNIFY is built up along the way and is used to make sure that later comparisons are consistent with bindings that were established earlier. In a compound expression such as F(A, B), the Op field picks out the function symbol F and the ARGS field picks out the argument list (A, B). Otherwise, fail.
Hard matching example

• To unify the grounded propositions with premises of the implication you need to solve a CSP!
• Colorable() is inferred iff the CSP has a solution
• CSPs include 3SAT as a special case, hence matching is NP-hard

\[
\begin{align*}
&\text{Diff}(wa,nt) \land \text{Diff}(wa,sa) \land \text{Diff}(nt,q) \land \\
&\text{Diff}(nt,sa) \land \text{Diff}(q,nsw) \land \text{Diff}(q,sa) \land \\
&\text{Diff}(nsw,v) \land \text{Diff}(nsw,sa) \land \text{Diff}(v,sa) \Rightarrow \\
&\text{Colorable}() \\
&\text{Diff}(Red,Blue) \quad \text{Diff}(Red,Green) \\
&\text{Diff}(Green,Red) \quad \text{Diff}(Green,Blue) \\
&\text{Diff}(Blue,Red) \quad \text{Diff}(Blue,Green)
\end{align*}
\]
**FOL (or FOPC) Ontology:**
What kind of things exist in the world?
What do we need to describe and reason about?
Objects --- with their relations, functions, predicates, properties, and general rules.
Summary

- First-order logic:
  - Much more expressive than propositional logic
  - Allows objects and relations as semantic primitives
  - Universal and existential quantifiers

- Syntax: constants, functions, predicates, equality, quantifiers

- Nested quantifiers

- Translate simple English sentences to FOPC and back

- Semantics: correct under any interpretation and in any world

- Unification: Making terms identical by substitution
  - The terms are universally quantified, so substitutions are justified.