First-Order Logic
Knowledge Representation

Reading: Chapter 8, 9.1-9.2, 9.5.1-9.5.5

FOL Syntax and Semantics read: 8.1-8.2
FOL Knowledge Engineering read: 8.3-8.5
FOL Inference read: Chapter 9.1-9.2, 9.5.1-9.5.5

(Please read lecture topic material before and after each lecture on that topic)
Outline

• Review --- Syntactic Ambiguity

• Using FOL
  – Tell, Ask

• Example: Wumpus world

• Deducing Hidden Properties
  – Keeping track of change
  – Describing the results of Actions

• Set Theory in First-Order Logic

• Knowledge engineering in FOL

• The electronic circuits domain
You will be expected to know

- Seven steps of Knowledge Engineering (R&N section 8.4.1)

- Given a simple Knowledge Engineering problem, produce a simple FOL Knowledge Base that solves the problem
• FOPC provides many ways to represent the same thing.
  • E.g., “Ball-5 is red.”
    – HasColor(Ball-5, Red)
      • Ball-5 and Red are objects related by HasColor.
    – Red(Ball-5)
      • Red is a unary predicate applied to the Ball-5 object.
    – HasProperty(Ball-5, Color, Red)
      • Ball-5, Color, and Red are objects related by HasProperty.
    – ColorOf(Ball-5) = Red
      • Ball-5 and Red are objects, and ColorOf() is a function.
    – HasColor(Ball-5(), Red())
      • Ball-5() and Red() are functions of zero arguments that both
        return an object, which objects are related by HasColor.
    – ...

• This can GREATLY confuse a pattern-matching reasoner.
  – Especially if multiple people collaborate to build the KB, and they
    all have different representational conventions.
Review --- Syntactic Ambiguity --- Partial Solution

- FOL can be TOO expressive, can offer TOO MANY choices

- Likely confusion, especially for **teams** of Knowledge Engineers

- Different team members can make different representation choices
  - E.g., represent “Ball43 is Red.” as:
    - a predicate (= verb)? E.g., “Red(Ball43)” ?
    - an object (= noun)? E.g., “Red = Color(Ball43))” ?
    - a property (= adjective)? E.g., “HasProperty(Ball43, Red)” ?

- **PARTIAL SOLUTION:**
  - An upon-agreed **ontology** that settles these questions
  - Ontology = what exists in the world & how it is represented
  - The Knowledge Engineering teams agrees upon an ontology BEFORE they begin encoding knowledge
Using FOL

• We want to TELL things to the KB, e.g.
  TELL(KB, \(\forall x, King(x) \Rightarrow Person(x)\))
  TELL(KB, King(John))

  These sentences are assertions

• We also want to ASK things to the KB,
  ASK(KB, \(\exists x, Person(x)\))

  these are queries or goals

The KB should return the list of x’s for which Person(x) is true:
\{x/John, x/Richard, ...\}
Knowledge engineering in FOL

1. Identify the task
2. Assemble the relevant knowledge
3. Decide on a vocabulary of predicates, functions, and constants
4. Encode general knowledge about the domain
5. Encode a description of the specific problem instance
6. Pose queries to the inference procedure and get answers
7. Debug the knowledge base
FOL Version of Wumpus World

- Typical percept sentence:
  Percept([Stench,Breeze,Glitter,None,None],5)

- Actions:
  Turn(Right), Turn(Left), Forward, Shoot, Grab, Release, Climb

- To determine best action, construct query:
  \[ \exists a \text{ BestAction}(a,5) \]

- ASK solves this and returns \{a/Grab\}
  - And TELL about the action.
Knowledge Base for Wumpus World

- Perception
  - $\forall s, g, x, y, t \ Percept([s, Breeze, g, x, y], t) \Rightarrow Breeze(t)$
  - $\forall s, b, x, y, t \ Percept([s, b, Glitter, x, y], t) \Rightarrow Glitter(t)$

- Reflex action
  - $\forall t \ Glitter(t) \Rightarrow BestAction(Grab, t)$

- Reflex action with internal state
  - $\forall t \ Glitter(t) \wedge \neg Holding(Gold, t) \Rightarrow BestAction(Grab, t)$

Holding(Gold,t) can not be observed: keep track of change.
Deducing hidden properties

Environment definition:

\[ \forall x,y,a,b \ Adjacent([x,y],[a,b]) \iff [a,b] \in \{[x+1,y], [x-1,y],[x,y+1],[x,y-1]\} \]

Properties of locations:

\[ \forall s,t \ At(Agent,s,t) \land Breeze(t) \Rightarrow Breezy(s) \]

Squares are breezy near a pit:

- **Diagnostic** rule---infer cause from effect
  \[ \forall s \ Breezy(s) \iff \exists r \ Adjacent(r,s) \land Pit(r) \]

- **Causal** rule---infer effect from cause (model based reasoning)
  \[ \forall r \ Pit(r) \Rightarrow [\forall s \ Adjacent(r,s) \Rightarrow Breezy(s)] \]
Keeping track of change

Facts hold in situations, rather than eternally.
E.g., $\text{Holding}(\text{Gold}, \text{Now})$ rather than just $\text{Holding}(\text{Gold})$

**Situation calculus** is one way to represent change in FOL:
  Adds a situation argument to each non-eternal predicate
  E.g., $\text{Now}$ in $\text{Holding}(\text{Gold}, \text{Now})$ denotes a situation

Situations are connected by the $\text{Result}$ function.
$\text{Result}(a, s)$ is the situation that results from doing $a$ in $s$.
Describing actions I

“Effect” axiom—describe changes due to action
\( \forall s \ AtGold(s) \Rightarrow Holding(Gold, Result(Grab, s)) \)

“Frame” axiom—describe non-changes due to action
\( \forall s \ HaveArrow(s) \Rightarrow HaveArrow(Result(Grab, s)) \)

Frame problem: find an elegant way to handle non-change
   (a) representation—avoid frame axioms
   (b) inference—avoid repeated “copy-overs” to keep track of state

Qualification problem: true descriptions of real actions require endless caveats—what if gold is slippery or nailed down or . . .

Ramification problem: real actions have many secondary consequences—what about the dust on the gold, wear and tear on gloves, . . .
Describing actions II

Successor-state axioms solve the representational frame problem

Each axiom is “about” a predicate (not an action per se):

\[ P \text{ true afterwards} \iff \text{[an action made } P \text{ true} } \]
\[ \quad \lor \quad P \text{ true already and no action made } P \text{ false] } \]

For holding the gold:

\[ \forall a, s \quad Holding(Gold, Result(a, s)) \iff \]
\[ \quad [(a = \text{Grab} \land \text{AtGold}(s)) \]
\[ \quad \lor (Holding(Gold, s) \land a \neq \text{Release})] \]
Set Theory in First-Order Logic

Can we define set theory using FOL?
- individual sets, union, intersection, etc

Answer is yes.

Basics:
- empty set = constant = \{ \} 
- unary predicate Set( ), true for sets
- binary predicates:
  - \( x \in s \) (true if \( x \) is a member of the set \( s \))
  - \( s_1 \subseteq s_2 \) (true if \( s_1 \) is a subset of \( s_2 \))
- binary functions:
  - intersection \( s_1 \cap s_2 \), union \( s_1 \cup s_2 \), adjoining \( \{ x | s \} \)
A Possible Set of FOL Axioms for Set Theory

The only sets are the empty set and sets made by adjoining an element to a set
\[ \forall s \, \text{Set}(s) \iff (s = \{\} ) \lor (\exists x, s_2 \, \text{Set}(s_2) \land s = \{x|s_2\}) \]

The empty set has no elements adjoined to it
\[ \neg \exists x, s \, \{x|s\} = \{\} \]

Adjoining an element already in the set has no effect
\[ \forall x, s \, x \in s \iff s = \{x|s\} \]

The only elements of a set are those that were adjoined into it. Expressed recursively:
\[ \forall x, s \, x \in s \iff [ \exists y, s_2 \, (s = \{y|s_2\} \land (x = y \lor x \in s_2))] \]
A Possible Set of FOL Axioms for Set Theory

A set is a subset of another set iff all the first set’s members are members of the 2nd set
\[ \forall s_1, s_2 \ s_1 \subseteq s_2 \iff (\forall x \ x \in s_1 \implies x \in s_2) \]

Two sets are equal iff each is a subset of the other
\[ \forall s_1, s_2 \ (s_1 = s_2) \iff (s_1 \subseteq s_2 \land s_2 \subseteq s_1) \]

An object is in the intersection of 2 sets only if a member of both
\[ \forall x, s_1, s_2 \ x \in (s_1 \cap s_2) \iff (x \in s_1 \land x \in s_2) \]

An object is in the union of 2 sets only if a member of either
\[ \forall x, s_1, s_2 \ x \in (s_1 \cup s_2) \iff (x \in s_1 \lor x \in s_2) \]
The electronic circuits domain

One-bit full adder

Possible queries:
- does the circuit function properly?
- what gates are connected to the first input terminal?
- what would happen if one of the gates is broken?
and so on
The electronic circuits domain

1. Identify the task
   - Does the circuit actually add properly?

2. Assemble the relevant knowledge
   - Composed of wires and gates; Types of gates (AND, OR, XOR, NOT)
   - Irrelevant: size, shape, color, cost of gates

3. Decide on a vocabulary
   - Alternatives:
     - Type($X_1$) = XOR (function)
     - Type($X_1$, XOR) (binary predicate)
     - XOR($X_1$) (unary predicate)
4. Encode general knowledge of the domain
   - \( \forall t_1, t_2 \text{ Connected}(t_1, t_2) \Rightarrow \text{Signal}(t_1) = \text{Signal}(t_2) \)
   - \( \forall t \text{ Signal}(t) = 1 \lor \text{Signal}(t) = 0 \)
   - \( 1 \neq 0 \)
   - \( \forall t_1, t_2 \text{ Connected}(t_1, t_2) \Rightarrow \text{Connected}(t_2, t_1) \)
   - \( \forall g \text{ Type}(g) = \text{OR} \Rightarrow \text{Signal}(\text{Out}(1,g)) = 1 \Leftrightarrow \exists n \text{ Signal}(\text{In}(n,g)) = 1 \)
   - \( \forall g \text{ Type}(g) = \text{AND} \Rightarrow \text{Signal}(\text{Out}(1,g)) = 0 \Leftrightarrow \exists n \text{ Signal}(\text{In}(n,g)) = 0 \)
   - \( \forall g \text{ Type}(g) = \text{XOR} \Rightarrow \text{Signal}(\text{Out}(1,g)) = 1 \Leftrightarrow \text{Signal}(\text{In}(1,g)) \neq \text{Signal}(\text{In}(2,g)) \)
   - \( \forall g \text{ Type}(g) = \text{NOT} \Rightarrow \text{Signal}(\text{Out}(1,g)) \neq \text{Signal}(\text{In}(1,g)) \)
The electronic circuits domain

5. Encode the specific problem instance

\[
\begin{align*}
\text{Type}(X_1) &= \text{XOR} & \text{Type}(X_2) &= \text{XOR} \\
\text{Type}(A_1) &= \text{AND} & \text{Type}(A_2) &= \text{AND} \\
\text{Type}(O_1) &= \text{OR} \\
\end{align*}
\]

\[
\begin{align*}
\text{Connected}(\text{Out}(1,X_1),\text{In}(1,X_2)) & \quad \text{Connected}(\text{In}(1,C_1),\text{In}(1,X_1)) \\
\text{Connected}(\text{Out}(1,X_1),\text{In}(2,A_2)) & \quad \text{Connected}(\text{In}(1,C_1),\text{In}(1,A_1)) \\
\text{Connected}(\text{Out}(1,A_2),\text{In}(1,O_1)) & \quad \text{Connected}(\text{In}(2,C_1),\text{In}(2,X_1)) \\
\text{Connected}(\text{Out}(1,A_1),\text{In}(2,O_1)) & \quad \text{Connected}(\text{In}(2,C_1),\text{In}(2,A_1)) \\
\text{Connected}(\text{Out}(1,X_2),\text{Out}(1,C_1)) & \quad \text{Connected}(\text{In}(3,C_1),\text{In}(2,X_2)) \\
\text{Connected}(\text{Out}(1,O_1),\text{Out}(2,C_1)) & \quad \text{Connected}(\text{In}(3,C_1),\text{In}(1,A_2))
\end{align*}
\]
6. Pose queries to the inference procedure
What are the possible sets of values of all the terminals for the adder circuit?

\[ \exists i_1, i_2, i_3, o_1, o_2 \text{ Signal(In}(1, C_1)) = i_1 \land \text{Signal(In}(2, C_1)) = i_2 \land \text{Signal(In}(3, C_1)) = i_3 \land \text{Signal(Out}(1, C_1)) = o_1 \land \text{Signal(Out}(2, C_1)) = o_2 \]

7. Debug the knowledge base
May have omitted assertions like \( 1 \neq 0 \)
Review --- Knowledge engineering in FOL

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Summary

- First-order logic:
  - Much more expressive than propositional logic
  - Allows objects and relations as semantic primitives
  - Universal and existential quantifiers
  - Syntax: constants, functions, predicates, equality, quantifiers

- Knowledge engineering using FOL
  - Capturing domain knowledge in logical form