## CS-171, Intro to A.I. — Mid-term Exam — Winter Quarter, 2012

YOUR NAME AND ID NUMBER: $\qquad$
YOUR ID: $\qquad$ ID TO RIGHT: $\qquad$ ROW: $\qquad$ NO. FROM RIGHT: $\qquad$

The exam will begin on the next page. Please, do not turn the page until told.
When you are told to begin the exam, please check first to make sure that you have all ten pages, as numbered 1-10 in the bottom-left corner of each page.

The exam is closed-notes, closed-book. No calculators, cell phones, electronics.
Please clear your desk entirely, except for pen, pencil, eraser, an optional blank piece of paper (for optional scratch pad use), and an optional water bottle.

This page summarizes the points available for each question so you can plan your time.

1. ( 12 pts total, 1 pt each ) LOGIC CONCEPTS.
2. ( 5 pts total, -1 for each error, but not negative) ALPHA-BETA PRUNING.
3. ( 5 pts total, 1 pt each) CONSTRAINT SATISFACTION PROBLEMS.
4. ( 10 pts total, 2 pts each) STATE-SPACE SEARCH.
5. ( 15 pts total, 1 pt each) AGENT/SEARCH CONCEPTS.
6. ( 5 pts total, -2 pts each wrong answer, but not negative) TASK ENVIRONMENT.
7. ( 10 pts total, -1 each wrong answer, but not negative) SEARCH PROPERTIES.
8. (10 pts total, -1 each error, but not negative) STATE SPACE SEARCH.
9. (10 pts total, -2 each error, but not negative) OPTIMALITY OF A* SEARCH.
10. (4 pts total, 1 pt each) RESOLUTION.
11. ( 10 pts total, 1 pt each) PROVE THAT THE UNICORN IS MAGICAL.
12. (4 pts total, -1 each error, but not negative) CONJUNCTIVE NORMAL FORM (CNF).
13. (12 pts total, $\mathbf{1} \mathbf{~ p t ~ e a c h ) ~ L O G I C ~ C O N C E P T S . ~ F o r ~ e a c h ~ o f ~ t h e ~ f o l l o w i n g ~ t e r m s ~ o n ~ t h e ~ l e f t , ~ w r i t e ~ i n ~}$ the letter corresponding to the best answer or the correct definition on the right. The first one is done for you as an example.

| A | Agent | A | Perceives environment by sensors, acts by actuators. |
| :--- | :--- | :--- | :--- |
|  | Syntax | B | Chain of inference rule conclusions leading to a desired sentence. |
|  | Semantics | C | Specifies all the sentences in a language that are well formed. |
|  | Entailment | D | Describes a sentence that is true in all models. |
|  | Sound | E | Stands for a proposition that can be true or false. |
|  | Complete | F | Represented as a canonical conjunction of disjunctions. |
|  | Propositional Symbol | G | Possible world that assigns TRUE or FALSE to each proposition. |
|  | Valid | H | Describes a sentence that is false in all models. |
|  | Satisfiable | I | Defines truth of each sentence with respect to each possible world. |
|  | Unsatisfiable | J | An inference procedure that derives only entailed sentences. |
|  | Proof | K | An inference procedure that derives all entailed sentences. |
|  | Model | L | The idea that a sentence follows logically from other sentences. |
|  | Conjunctive Normal Form | M | Describes a sentence that is true in some model. |

2. ( $\mathbf{5} \mathbf{p t s}$ total, $\mathbf{- 1}$ for each error, but not negative) ALPHA-BETA PRUNING. The game tree below illustrates a position reached in the game. It is Max's turn to move. Inside each leaf node is the estimated score of that resulting position returned by the heuristic static evaluator.
CROSS OUT EACH LEAF NODE THAT WILL BE PRUNED BY ALPHA-BETA PRUNING. You do not need to indicate the branch node values.

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3. (5 pts total, 1 pt each) CONSTRAINT SATISFACTION PROBLEMS. This problem asks about the Map Coloring Problem. Each region must be colored one of Red (R), Green (G), or Blue (B). Neighboring regions must be a different color. The map (left) and constraint graph (right) are below.


3a. (1 pt) FORWARD CHECKING. Consider the partial assignment below. Variable B has been assigned value R as shown. Cross out all values that would be eliminated by Forward Checking (FC) after the assignment to variable B.

| A | B | C | D | E | F |
| :---: | :---: | :---: | :---: | :---: | :---: |
| R G B | R | R G B | R G B | R G B | R G B |

3b. (1 pt) ARC CONSISTENCY. Consider the partial assignment below. Variables A and B have been assigned values as shown. Cross out all other values that would be eliminated by Arc Consistency (AC, also called AC-3 in your book).

| A | B | C | D | E | F |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $R$ | $G$ | R G B | R G B | R G B | R G B |

3c. (1 pt) MINIMUM-REMAINING-VALUES HEURISTIC. Consider the partial assignment below. Variable A is already assigned value R, and Arc Consistency is already done. List all unassigned variables that might possibly be selected by the Minimum-Remaining-Values (MRV) Heuristic:
$\qquad$ .

| A | B | C | D | E | F |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $R$ | $G B$ | R G B | G B | R G B | R G B |

3d. (1 pt) DEGREE HEURISTIC. Consider the partial assignment below. (It is the same assignment as in problem 3c above.) List all unassigned variables that might possibly be selected by the Degree Heuristic: $\qquad$ .

| $A$ | $B$ | $C$ | $D$ | $E$ | $F$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $R$ | $G B$ | $R G B$ | $G B$ | $R G B$ | $R G B$ |

3e. (1 pt) MIN-CONFLICTS HEURISTIC. Consider the complete but inconsistent assignment below. E has just now been selected to be assigned a new value. List all new values that might be chosen below for E by the Min-Conflicts Heuristic? $\qquad$ .

| A | B | C | D | E | F |
| :---: | :---: | :---: | :---: | :---: | :---: |
| R | G | B | R | $?$ | G |

4. (10 pts total, 2 pts each) STATE-SPACE SEARCH. Execute Tree Search through this graph (do not remember visited nodes, so repeated nodes are possible). It is not a tree, but pretend you don’t know that. Step costs are given next to each arc, and heuristic values are given next to each node (as $\mathrm{h}=\mathrm{x}$ ). The successors of each node are indicated by the arrows out of that node. (Note: $\mathbf{C}$ is a successor of itself).

For each search strategy below, indicate the order in which nodes are expanded (i.e., to expand a node means that its children are generated), ending with the goal node that is found.

4.a. (2 pts, -1 for each wrong answer, but not negative) UNIFORM COST SEARCH.
4.b. ( 2 pts, $\mathbf{- 1}$ for each wrong answer, but not negative) GREEDY BEST-FIRST SEARCH.
4.c ( $\mathbf{2} \mathbf{~ p t s , ~} \mathbf{- 1}$ for each wrong answer, but not negative) ITERATIVE DEEPENING SEARCH.
4.d. (2 pts, -1 for each wrong answer, but not negative) A* SEARCH.
4.e. ( $\mathbf{2}$ pts, $\mathbf{- 1}$ for each wrong answer, but not negative) OPTIMALITY.

Did Uniform Cost Search find the optimal goal? $\qquad$ Why or why not?
$\qquad$ Why or why not?
5. (15 pts total, 1 pt each) AGENT/SEARCH CONCEPTS. For each of the following terms on the left, write in the letter corresponding to the best answer or the correct definition on the right. The first one is done for you as an example.

| A | Agent | A | Perceives environment by sensors, acts by actuators |
| :--- | :--- | :--- | :--- |
|  | Percept | B | All states reachable from the initial state by a sequence of actions |
|  | Performance Measure | C | Guaranteed to find a solution if one is accessible |
|  | Rational Agent | D | Process of removing detail from a representation |
|  | State Space | E | Maximum number of successors of any node |
|  | Search Node | F | Set of all leaf nodes available for expansion at any given time |
|  | Link between nodes | G | Estimates cost of cheapest path from current state to goal state |
|  | Path | H | Guaranteed to find lowest cost among all accessible solutions |
|  | Abstraction | I | Represents a state in the state space |
|  | Optimal Search | J | Sequence of states connected by a sequence of actions |
|  | Complete Search | K | Agent’s perceptual inputs at any given instant |
|  | Expand a state | L | Agent that acts to maximize its expected performance measure |
|  | Frontier | M | Apply each legal action to a state, generating a new set of states |
|  | Search Strategy | N | Represents an action in the state space |
|  | Branching Factor | O | How a search algorithm chooses which node to expand next |
|  | Heuristic Function | P | Evaluates any given sequence of environment states for utility |

6. (5 pts total, -2 pts each wrong answer, but not negative) TASK ENVIRONMENT. A task environment is a set of four things, with acronym PEAS. Fill in the blanks with the PEAS components.
P
E $\qquad$ A
S

## 7. (10 pts total, $\mathbf{- 1}$ each wrong answer, but not negative) SEARCH PROPERTIES.

Fill in the values of the four evaluation criteria for each search strategy shown. Assume a tree search where $b$ is the finite branching factor; d is the depth to the shallowest goal node; m is the maximum depth of the search tree; $\mathrm{C}^{*}$ is the cost of the optimal solution; step costs are identical and equal to some positive $\varepsilon$; and in Bidirectional search both directions use breadth-first search.

Note that these conditions satisfy all of the footnotes of Fig. 3.21 in your book.

| Criterion | Complete? | Time complexity | Space complexity | Optimal? |
| :--- | :--- | :--- | :--- | :--- |
| Breadth-First |  |  |  |  |
| Uniform-Cost |  |  |  |  |
| Depth-First |  |  |  |  |
| Iterative Deepening |  |  |  |  |
| Bidirectional <br> (if applicable) |  |  |  |  |

8. (10 pts total, -1 each error, but not negative) STATE SPACE SEARCH.

A man is traveling to market with a fox, a goose, and a bag of oats. He comes to a river. The only way across the river is a boat that can hold the man and exactly one of the fox, goose or bag of oats. The fox will eat the goose if left alone with it, and the goose will eat the oats if left alone with it. How can the man get all his possessions safely across the river?

We will call states where something gets eaten "forbidden" and consider them as part of the state space, but from which there is no return; i.e., there is no way to go to any other state (including back to the previous state) from such a forbidden state. All other states are "allowed."

One way to represent this problem is as a state vector with components ( $M, F, G, O$ ), where $\mathrm{M}=\mathrm{man}, \mathrm{F}=$ fox, $\mathrm{G}=$ goose, and $\mathrm{O}=$ oats are binary variables that are 0 if the man/fox/goose/oats are on the near bank, and 1 if on the far bank. Note that the boat is always on the same side of the river as the man. Thus the start state is $(0,0,0,0)$ and the single goal state is $(1,1,1,1)$.
8.a. (1 pt) How many total states (both allowed and forbidden) in this state space?
8.b. (1 pt) What is the maximum branching factor counting BOTH allowed and forbidden states as children?
8.c. (8 pts, -1 for each error, but not negative) Draw the state space considering only ALLOWED states. Represent an allowed state as a circle enclosing its binary vector (M,F,G,O). Connect states that are possible successors of each other. The start and goal states are shown. The first one is done for you as an example.

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9. ( 10 pts total, -2 each error, but not negative) OPTIMALITY OF A* SEARCH. You know that $\mathrm{A}^{*}$ tree search with an admissible heuristic is an optimal search strategy. Recall that:

$$
\begin{aligned}
& g(n)=\text { true path cost so far } \\
& h(n)=\text { estimated optimal cost from } \mathrm{n} \text { to goal } \leq \text { true optimal cost } \\
& f(n)=g(n)+h(n)=\text { estimated total optimal path cost } \leq \text { true total optimal cost }
\end{aligned}
$$

The following is a proof that A* tree search (queue sorted by $f(n)$ ) is optimal if the heuristic is admissible. The lines of the proof have been labeled A through G. Unfortunately, they have been scrambled.

Let $n g$ be the first goal node popped off the queue. Let no be any other node still on the queue. We wish to prove that the path through no can never be extended to a path to any goal node that costs less than the path to $n g$ that we just found.

A: true total cost of path to ng

| B: | $\leq f(n o) \quad / /$ because queue is sorted by $f$ and $n g$ came off queue first |
| :--- | :--- |
| C: | $\leq g(n o)+$ true optimal cost to goal from no $\quad / /$ because $h$ is admissible |
| D: | $=f(n g) \quad /$ by definition of $f$, with $h(n g)=0$ because $n g$ is a goal node |
| E: | $=g(n o)+h(n o) \quad / /$ because $n g$ represents a complete path |
| F: | $=g(n g) \quad$ refinite of |
| G: | $=$ true total optimal path cost through no |

Fill in the blanks with the letters B, C, D, E, F , and G to prove that the true total cost of the path to ng is less than or equal to the true total optimal path cost through no. The first and last letters, A and G, have been done for you as an example.
$\mathrm{A} \quad \ldots \quad$ _ $\quad$ G. .
10. (4 pts total, $\mathbf{1} \mathbf{~ p t ~ e a c h ) ~ R E S O L U T I O N . ~ A p p l y ~ r e s o l u t i o n ~ t o ~ e a c h ~ o f ~ t h e ~ f o l l o w i n g ~ p a i r s ~ o f ~}$ clauses, then simplify. Write your answer in Conjunctive Normal Form (CNF).
10.a. (1 pt) (A B $\neg C D)(A \neg D E F)$. $\qquad$
10.b. (1 pt) (A B $\neg C D)(\neg A)$. $\qquad$
10.c. (1 pt) ( $\neg \mathrm{C})(\mathrm{C})$.
10.d. (1 pt) (A B $\neg \mathrm{C} D)(\mathrm{A} \mathrm{C} \neg \mathrm{D} E \mathrm{~F})$.

## 11. (10 pts total, 1 pt each) PROVE THAT THE UNICORN IS MAGICAL.

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

Use these propositional variables:

$$
\begin{array}{lll}
\mathbf{Y}=\text { unicorn is m} \mathbf{Y} \text { thical } & \mathbf{R}=\text { unicorn is moRtal } & \mathbf{M}=\text { unicorn is a maMmal } \\
\mathbf{H}=\text { unicorn is Horned } & \mathbf{G}=\text { unicorn is maGical } &
\end{array}
$$

11.a. (5 pts total, pt each) Convert the English into propositional logic implicative form and conjunctive normal form (CNF). The first one is done for you as an example. (Note: "immortal" means "not mortal.")
11.a.1. If the unicorn is mYthical, then it is not moRtal.

S1: Implicative $\qquad$ . $\qquad$
CNF
11.a.2. (1 pt) If the unicorn is not mYthical, then it is moRtal.

S2: Implicative $\qquad$ .

CNF $\qquad$
11.a.3. ( $\mathbf{1} \mathbf{~ p t}$ ) If the unicorn is not mYthical, then it is a maMmal.

S3: Implicative $\qquad$ .

CNF $\qquad$
11.a.4. ( $\mathbf{1} \mathbf{~ p t}$ ) If the unicorn is not moRtal, then it is Horned.

S4: Implicative $\qquad$ .

CNF $\qquad$
11.a.5. (1 pt) If the unicorn is a maMmal, then it is Horned.

S5: Implicative $\qquad$ .

CNF $\qquad$
11.a.6. (1 pt) The unicorn is maGical if it is Horned. (same: "If the unicorn is Horned, then it is maGical.")

S6: Implicative $\qquad$ -

CNF $\qquad$
11.b. (5 pts total, pt each) Resolution Theorem Proving. Use the conjunctive normal form (CNF) expressions from 11.a above to prove that the unicorn is magical. The first and last steps are done for you. Express your answers in CNF.
11.b.1. The negated goal is S7.

S7: $\qquad$ .
11.b.2. (1 pt) Resolve S 6 and S 7 to give S 8 .

S8: $\qquad$ .
11.b.3. (1 pt) Resolve S5 and S8 to give S9.

> Here you will get full credit if you do the resolution steps correctly based on your answers for the two resolved sentences (even if those sentences were not correct). I.e., this question asks only that you do the resolution step correctly, regardless of content.

S9: $\qquad$ .
11.b.4. (1 pt) Resolve S4 and S8 to give S10.

S10: $\qquad$ .
11.b.5. (1 pt) Resolve S3 and S9 to give S11.

S11: $\qquad$ .
11.b.6. (1 pt) Resolve S1 and S11 to give S12.

S12: $\qquad$ .
11.b.7. Resolve S10 and S12 to give the empty clause, thus proving the goal sentence is true.

S13: $\qquad$ .
12. (4 pts total, $\mathbf{- 1}$ each error, but not negative) CONJUNCTIVE NORMAL FORM (CNF). Convert the following logical sentence to Conjunctive Normal Form. Show your work.

$$
\mathbf{B} \Leftrightarrow(\mathbf{P} \vee \mathbf{Q})
$$

