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YOUR ID: $\qquad$ ID TO RIGHT: $\qquad$ ROW: $\qquad$ SEAT: $\qquad$

1. (50 pts total) One Fish, Two Fish, Red Fish, Blue Fish. (With apologies to Dr. Suess.)

Decision Tree Classifier Learning. You are a robot in the aquarium section of a pet store, and must learn to discriminate Red fish from Blue fish. Unfortunately, your vision sensors are in Black \& White, but Red fish have the same gray-scale tone as Blue fish. So, you must learn to discriminate them by body parts. You choose to learn a Decision Tree classifier. You are given the following examples:

| Example | Fins | Tail | Body | Class |
| :--- | :--- | :--- | :--- | :--- |
| Example \#1 | Thin | Small | Slim | Red |
| Example \#2 | Wide | Large | Slim | Red |
| Example \#3 | Thin | Large | Slim | Red |
| Example \#4 | Wide | Small | Medium | Red |
| Example \#5 | Thin | Small | Medium | Blue |
| Example \#6 | Wide | Large | Fat | Blue |
| Example \#7 | Thin | Large | Fat | Blue |
| Example \#8 | Wide | Small | Fat | Blue |

Unfortunately, your textbook uses an inconsistent notation to refer to values of attributes. In Chap. 13, values of attributes (= random variables) are lower-case (see Section 13.2.2). In Chap. 18, values of attributes are upper-case (see Fig. 18.3). Here, since we are in the machine learning part of the course, we will follow Chap. 18 and use upper-case values of attributes. Please do not be confused.

1a. (20 pts) Which attribute would information gain choose as the root of the tree?

1b. (10 pts) Draw the decision tree that would be constructed by recursively applying information gain to select roots of sub-trees, as in the Decision-Tree-Learning algorithm.

Classify these new examples as Red or Blue using your decision tree above.
1c. (10 pts) What class is [Fins=Thin, Tail=Small, Body=Fat]? $\qquad$
1d. (10 pts) What class is [Fins=Wide, Tail=Large, Body=Medium]? $\qquad$
2. (50 pts total) One Fish, Two Fish, Red Fish, Blue Fish. (With apologies to Dr. Suess.) Naïve Bayes Classifier Learning. You are a robot in the aquarium section of a pet store, and must learn to discriminate Red fish from Blue fish. Unfortunately, your vision sensors are in Black \& White, but Red fish have the same gray-scale tone as Blue fish. So, you must learn to discriminate them by body parts. You choose to learn a Naïve Bayes classifier. You are given the following examples (these examples are different from the examples that were given in problem \#1, above):

| Example | Fins | Tail | Body | Class |
| :--- | :--- | :--- | :--- | :--- |
| Example \#1 | Thin | Large | Thin | Red |
| Example \#2 | Wide | Small | Thin | Red |
| Example \#3 | Wide | Large | Fat | Red |
| Example \#4 | Wide | Large | Fat | Red |
| Example \#5 | Thin | Small | Thin | Blue |
| Example \#6 | Thin | Large | Fat | Blue |
| Example \#7 | Wide | Small | Fat | Blue |
| Example \#8 | Thin | Small | Thin | Blue |

Unfortunately, your textbook uses an inconsistent notation to refer to values of attributes. In Chap. 13, values of attributes (= random variables) are lower-case (see Section 13.2.2). In Chap. 18, values of attributes are upper-case (see Fig. 18.3). Here, since we are in the machine learning part of the course, we will follow Chap. 18 and use upper-case values of attributes. Please do not be confused.
Recall that Bayes' rule allows you to rewrite the conditional probability of the class given the attributes as the conditional probability of the attributes given the class. As usual, $\alpha$ is a normalizing constant that makes the likelihoods (unnormalized probabilities) sum to one. Thus, we may ignore the repeated denominator $\mathbf{P}$ (Fins, Tail, Body), because it is constant for all classes. Using Bayes' Rule, we rewrite: $\mathbf{P}$ (Class | Fins, Tail, Body) $=\alpha \mathbf{P}$ (Fins, Tail, Body | Class) P(Class)
2a. (10 pts) Now assume that the attributes (Fins, Tail, and Body) are conditionally independent given the Class. Rewrite the expression above, using this assumption of conditional independence (i.e., rewrite it as a Naïve Bayes Classifier expression).
$\alpha$ P(Fins, Tail, Body | Class) $P($ Class $)=\alpha$
2b. (20 pts total; -2 for each wrong answer, but not negative) Fill in numerical values for the following expressions. Leave your answers as simplified common fractions (e.g., 1/4, 3/5).
$P($ Class=Red $)=$ $\qquad$
$P($ Fins=Thin | Class=Red $)=$ $\qquad$
$P($ Fins=Wide | Class=Red $)=$ $\qquad$
$P($ Tail=Large | Class=Red $)=$ $\qquad$
P(Tail=Small | Class=Red)= $\qquad$
$P($ Body $=$ Thin $\mid$ Class=Red $)=$ $\qquad$
$P($ Body=Fat | Class=Red $)=$ $\qquad$
2c. (20 pts total, 10 pts each) Consider a new example (Fins=Wide ^ Tail=Large ^ Body=Thin). Write these class probabilities as the product of $\alpha$ and common fractions from above. You do not need to produce an actual final number; only an expression that will evaluate to the right answer.
2.c.i (10 pts) P(Class=Red | Fins=Wide ^ Tail=Large ^ Body=Thin)

$$
=\underline{\alpha}
$$

2.c.ii (10 pts) P(Class=Blue | Fins=Wide ^ Tail=Large ^ Body $=$ Thin)
$=\underline{\alpha}$

