1. (5 pts each, 30 pts total) Mark the following reasoning patterns as S (= sound, carries true premises to true conclusions) or U (= unsound, may carry true premises to false conclusions). Premises are shown above the line, conclusions below the line. Here, “⇒” means “implies” and “¬” means “not.” The first one is done for you as an example.

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<td>a. S</td>
<td>P ⇒ Q</td>
<td>P</td>
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<td>c. U</td>
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<td>e. S</td>
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<td>g. U</td>
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2. (5 pts each, 40 pts total) In each of the following, $KB$ is a set of sentences, $\emptyset$ is the empty set of sentences, and $S$ is a single sentence. Recall $|= \text{ means “entails” and } |\!-\!| \ \text{means “derives,” }$ where $|\!-\!|_i \text{ means “inference procedure } i \text{ derives.” }$ Use these keys:

- Snd = Sound.
- Unsnd = Unsound.
- C = Complete.
- I = Incomplete.
- V = Valid.
- Sat = Satisfiable.
- Unsat = Unsatisfiable.
- N = None of the above.

For each blank below, write in the key above that best corresponds to the correct term.

(a) Suppose some inference procedure $i$ has the property, that for some $KB$ and some $S$, $KB |= S$ but not $KB |\!-\!|_i S$. Then the inference procedure $i$ is **I**.

(b) Let $S$ be given in advance. Suppose that for some $KB_1$, $KB_1 |= S$; but that for some other $KB_2$, $KB_2 |= \neg S$. Then $S$ is **Sat**.

(c) Suppose some inference procedure $i$ has the property, that for any $KB$ and any $S$, whenever $KB |= S$ then $KB |\!-\!|_i S$. Then the inference procedure $i$ is **C**.

(d) Suppose inference procedure $i$ has the property, that for some $KB$ and some $S$, $KB |\!-\!|_i S$ but not $KB |= S$. Then the inference procedure $i$ is **Unsnd**.

(e) Let $S$ be given in advance. Suppose that $\emptyset |= S$. Then $S$ is **V**.

(f) Suppose some inference procedure $i$ has the property, that for any $KB$ and any $S$, whenever $KB |\!-\!|_i S$ then $KB |= S$. Then the inference procedure $i$ is **Snd**.

(g) Suppose that $KB |= S$, then the sentence $(KB \Rightarrow S)$ is **V**.

(h) Suppose that $KB |= S$, then the sentence $(KB \text{ and } \neg S)$ is **Unsat**.
3. Consider the KB shown below.

a. (5 pts each, 15 pts total) Translate the following KB into Conjunctive Normal Form. The first one is done for you as an example (it was already in Conjunctive Normal Form ;-) ).

A. \( P \lor R \)

\[ P \lor R \]

B. \( Q \implies S \)

\[ \neg Q \lor S \]

C. \( P \implies Q \)

\[ \neg P \lor Q \]

D. \( R \implies S \)

\[ \neg R \lor S \]

b. (15 pts total, -5 for each wrong step, but not negative. The order may vary, if proof is correct.) Write a complete resolution proof that \( KB \models S \). Show the two clauses that you resolve in front of the symbol \( |- \), and the resulting clause after \( |- \). You may not require all of the lines provided. The sentence labeled “E.” adds the negated goal. The first one is done for you as an example.

E. \( \neg S \)

(a) \( \neg \neg S \), \( \neg \neg Q \lor S \), \( |- \), \( \neg \neg \neg Q \).

(b) \( \neg \neg Q \), \( \neg \neg \neg P \lor Q \), \( |- \), \( \neg \neg \neg P \).

(c) \( \neg \neg P \), \( \neg \neg \neg P \lor R \), \( |- \), \( \neg \neg \neg R \).

(d) \( \neg \neg R \), \( \neg \neg \neg R \lor S \), \( |- \), \( \neg \neg \neg S \).

(e) \( \neg \neg S \), \( \neg \neg \neg S \), \( |- \).

Other proofs are fine if correct. For example, at step (d) above you could have resolved with \( \neg S \):

(d) \( \neg \neg S \), \( \neg \neg \neg R \lor S \), \( |- \), \( \neg \neg \neg R \).

(e) \( \neg \neg R \), \( \neg \neg \neg R \lor S \), \( |- \).