CS 177, Sample Midterm Questions

Applications of Probability in Computer Science: Winter 2007

Problem 1

Answer each of the following questions, where X and Y are two discrete-valued random variables, with X taking m values $x_1, ..., x_m$, and Y taking n values $y_1, , ..., y_n$. You can use lower case x_i and y_j to denote generic values of variables X and Y respectively.

1. Define precisely $E[X]$, the expected value of X.
2. Use Bayes rule to write down an expression for $P(y_j|x_i)$.
3. Let Z be a 3rd variable taking values $z_1, \ldots, z_k$. Show (using 1 or more equations) how to compute $P(z_s|x_i, y_j)$, assuming that you are given the joint distribution $P(x_i, y_j, z_s)$ for all values x_i, y_j, z_s.
4. State precisely a set of necessary conditions for X and Y to be independent.
5. If we assume X and Y are independent (just for this part of the problem), precisely how many separate probability numbers are required to specify the joint probability mass function $P(X, Y)$. If we don't assume independence, how many probability numbers are needed?

Problem 2

You are a computer consultant who runs a very successful series of 1-day workshops on JAVA programming techniques. For each individual that registers there is a 3% chance that they will not show up (probability 0.03). Assume decisions are made independently by each registrant. The conference room where you hold the course can seat 100 people. If you decide to accept reservations for 104 people what is the probability that there will be a seat available for all of the registered people who show up?

Problem 3

A professor randomly selects 1 question to grade out of 8 questions on a homework

1. a student has answered 6 questions. What is the probability that none of the students' questions are graded?

2. a student answered k questions, where k varies from 1 to 8. Derive a general expression (i.e., a mathematical expression involving k) for the probability that none of the students' questions are graded.

3. A different professor grades 2 questions out of 8. What is the probability that a student who answered 6 questions out of the 8 will have none of those questions graded? what is the probability that a student will have only 1 question graded? what is the probability they will have 2 questions graded?

4. Solve part 3 for the general case where the student answered k questions, where k = 1, 2, ...8. Again your answers should be mathematical expressions expressed as a function of k.

Problem 4

Consider the following very simple Markov model for an e-commerce Web-site with 4 states: S for the start state, B for a browsing state, D for a check-out state, and E for an end-state.

- From the start state S the user always goes to state B.
- Given that a user is in state B, the user next goes to state D with probability 0.1, to state E with probability 0.2, and has a 0.7 probability of staying in state B.
- Given that a user is in state D, the user next goes to state B with probability 0.2, to state E with probability 0.7, and has a 0.1 probability of staying in state D.
- The end-state is an "absorbing" state—once the user reaches this state they stay there forever.

1. Define the transition matrix P for the corresponding Markov chain (where row 1 =S, row 2 = B, row 3 = D, row 4 = E).
2. If a user is in state S at time t, what is the probability of being in State E at time $t + 3$
3. What is the probability that a user makes a purchase (i.e., ever visits state D)?