Section 1.1, problem 62

Let’s denote:

\[ K = \text{“Kevin is chatting”} \]
\[ A = \text{“Abby is chatting”} \]
\[ H = \text{“Heather is chatting”} \]
\[ R = \text{“Randy is chatting”} \]
\[ V = \text{“Vijay is chatting”} \]

Here’s what we know about the world (in the order as they are stated in this exercise):

1. \( K \lor H \)
2. \( R \oplus V \)
3. \( A \rightarrow R \)
4. \((V \land K) \lor (\neg V \land \neg K)\)
5. \( H \rightarrow (A \land K) \)

So how do we know which combination is true? We have five variables here and 5 logical formulas which impose relations between these variables. So what we can do is to do a series of transformations which would simplify these relations:

2’. We can replace (2) by equation \( R = \neg V \).

3’. Therefore we can replace (3) by \( A \rightarrow \neg V \).

4’. Also we can replace (4) by an equivalent expression \( V \leftrightarrow K \)

Now let’s pick some variable on which lots of other variables seem to depend. For example pick \( H \), set it to true, and see what happens to the other variables:

If \( H \) then by (5) we immediately get \( A \) and \( K \). (1) is true. By (3’) from \( A \) we get \( \neg V \). However, by (4’) from \( \neg V \) we get \( \neg K \). But that’s a contradiction, because we cannot have both \( K \) and \( \neg K \).

What does this mean? Well, it means that \( H \) cannot be true, because that assumption led to a contradiction which violated the set of facts 1-5 we were told about the world. (In particular it led to a contradictory statement that \( K \land \neg K \). Therefore we can conclude that \( \neg H \). Assuming \( \neg H \), by (1) we get \( K \). By (4’) we get \( V \). By (3’) we get \( \neg A \). By (2’) we get \( \neg R \). And the only constraint left, (5), is satisfied because \( \neg H \).

This leads us to a solution \((K, \neg A, \neg H, \neg R, V)\) which, you can check, does satisfy all the criteria 1-5.

It is also a unique solution because we showed two things: First, that \( H \) cannot happen. Second, that if \( \neg H \) then the other four variables have uniquely assigned values, which are all a consequence of \( \neg H \) and facts 1-5. Therefore this is also the only solution.